

# **BARBADOS WORKSHOP**

## **HARMONIC ANALYSIS AND TIDE PREDICTION**

SILVIA COSTA

## CONTENS

- A. KEY CONSTITUENTS
- B. TIDAL REGIME
- C. AMPHIDROME SYSTEMS
- D. COTIDAL AND CORANGES MAPS

## A) KEY CONSTITUENTS

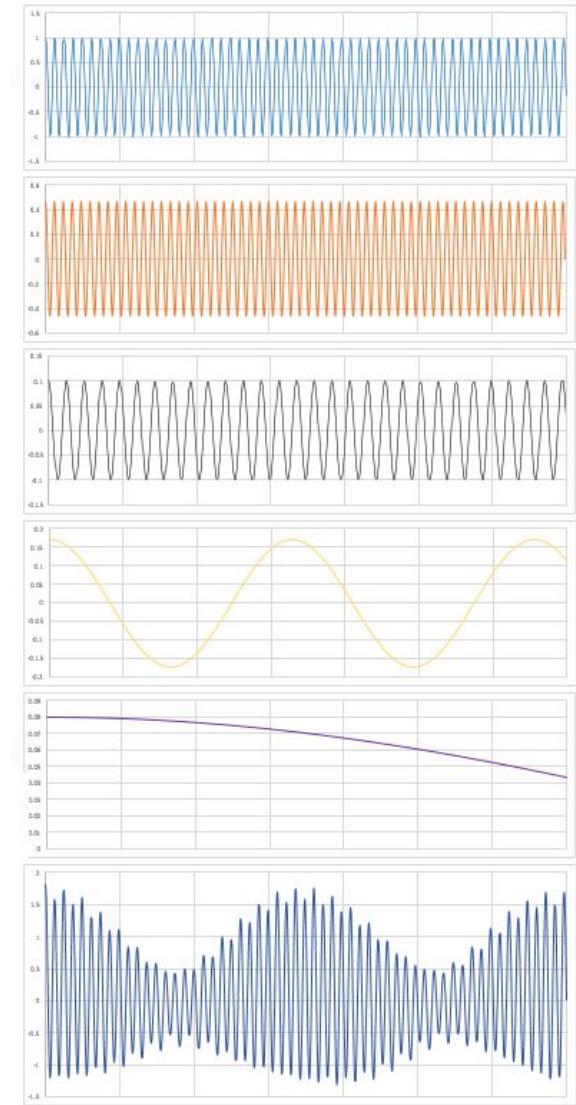
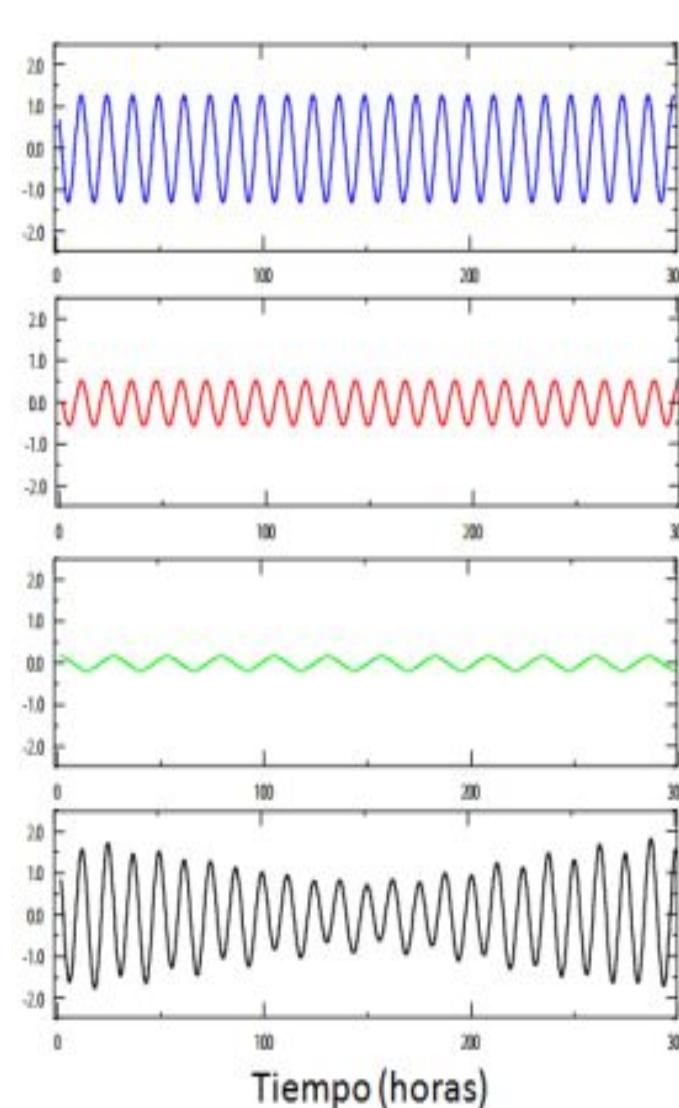
IF YOU HAVE A KEY.. YOU CAN OPEN A NEW WORLD

WE SAID.. THE TIDE IS A  
LONG WAVE...

WE CAN ASSUME THAT WE  
CAN APPLY ALL PROPIERTIES  
AND MATH OF WAVES



I CAN SUME WAVES UNTIL I  
HAVE A NEW WAVES NEAR  
THE REAL WAVE (TIDE  
RECORD)



## IN MATHEMATICAL LANGUAGE....

$$\eta(\tau) = a_0 + \sum_{n=1}^k a_n * \cos(\omega_n t + \alpha_n)$$

Tide height at an instant  
t

Amplitude of the  
constant

Constant phase

Letter related  
to the  
phenomenon

Mean level  
amplitude

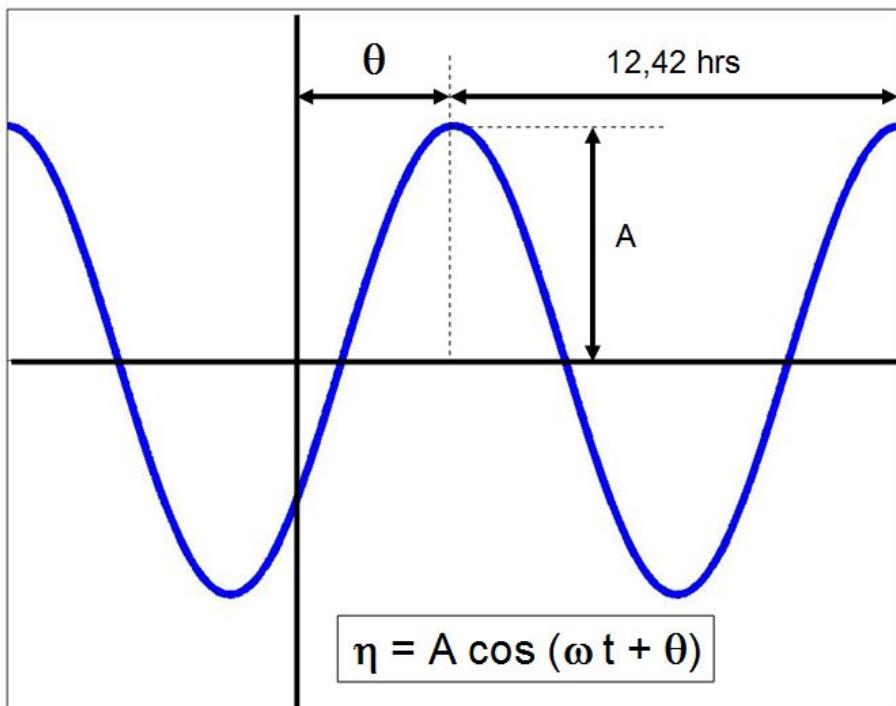
Angular frequency  
of the constant.  
KNOWN VALUE

**M<sub>2</sub>**

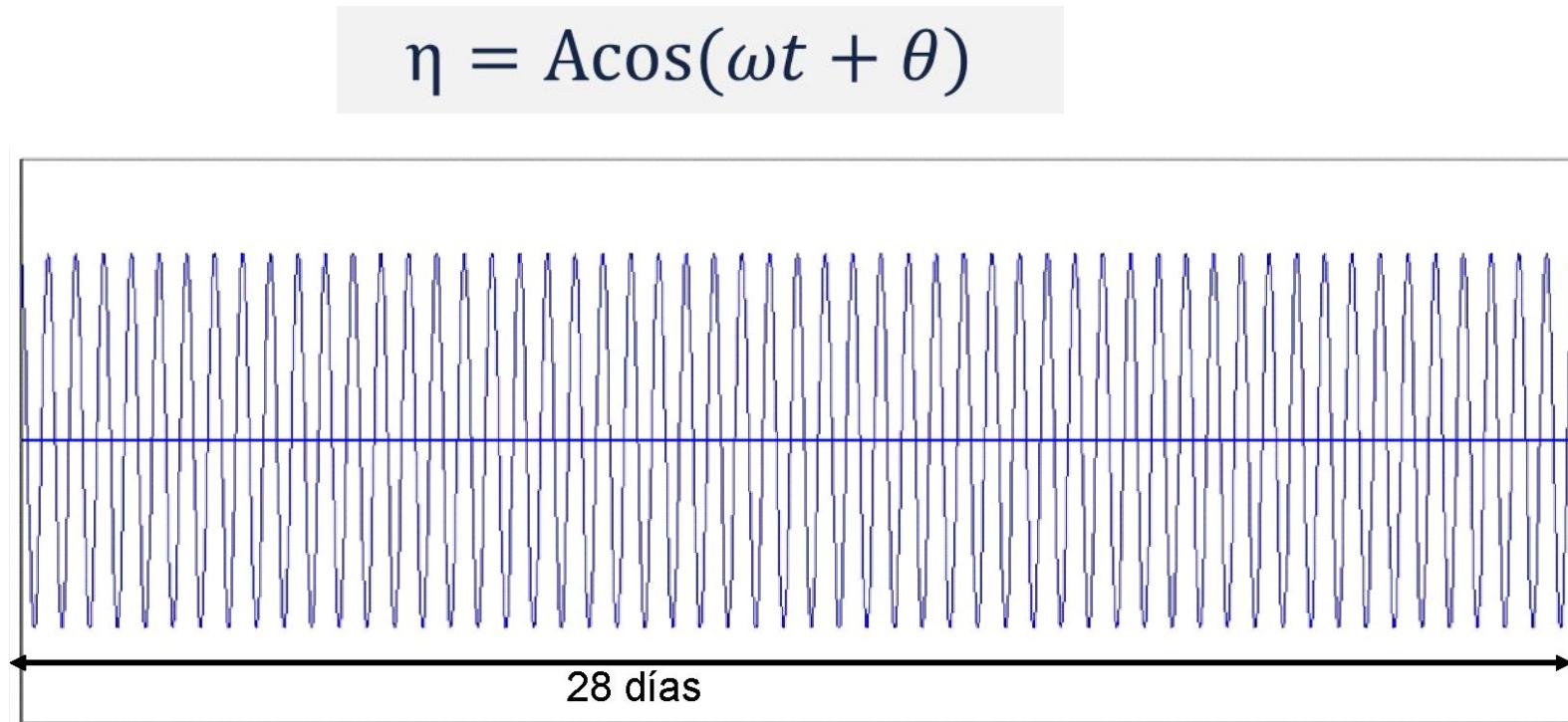
Harmonic Constituent

Number  
related to the  
period

LET'S START BY APPLYING THE SIMPLEST THING... ONLY THE ROTATION OF THE MOON AND THE EARTH AROUND THE CENTER OF MASS EXISTS



$$\eta = A \cos(\omega t + \theta)$$



PLAYING WITH MATHS.. AND KNOWING THAT MY AMPLITUDE CHANGES ALONG THE REGISTER...

$$\text{New amplitud} = A[1 + A_a \cos(\omega_a t + \theta_a)]$$

$$\eta = A[1 + A_a \cos(\omega_a t + \theta_a)] \cos(\omega t + \theta)$$

$$\eta = A \cos(\omega t + \theta) + A * A_a \cos(\omega_a t + \theta_a) \cos(\omega t + \theta)$$

→  $\cos a \cos b = \frac{1}{2} [\cos(a + b) + \cos(a - b)]$

$$\eta = A * \cos(\omega t + \theta) + A * A_a \cos(\omega_a t + \theta_a) \cos(\omega t + \theta)$$

$$\eta = A * \cos(\omega t + \theta) + \frac{1}{2} A * A_a * [\cos((\omega + \omega_a)t + (\theta + \theta_a)) + \cos((\omega - \omega_a)t + (\theta - \theta_a))]$$

$$\eta = A * \cos(\omega t + \theta) + \frac{1}{2} A * A_a * \cos((\omega + \omega_a)t + (\theta + \theta_a))$$

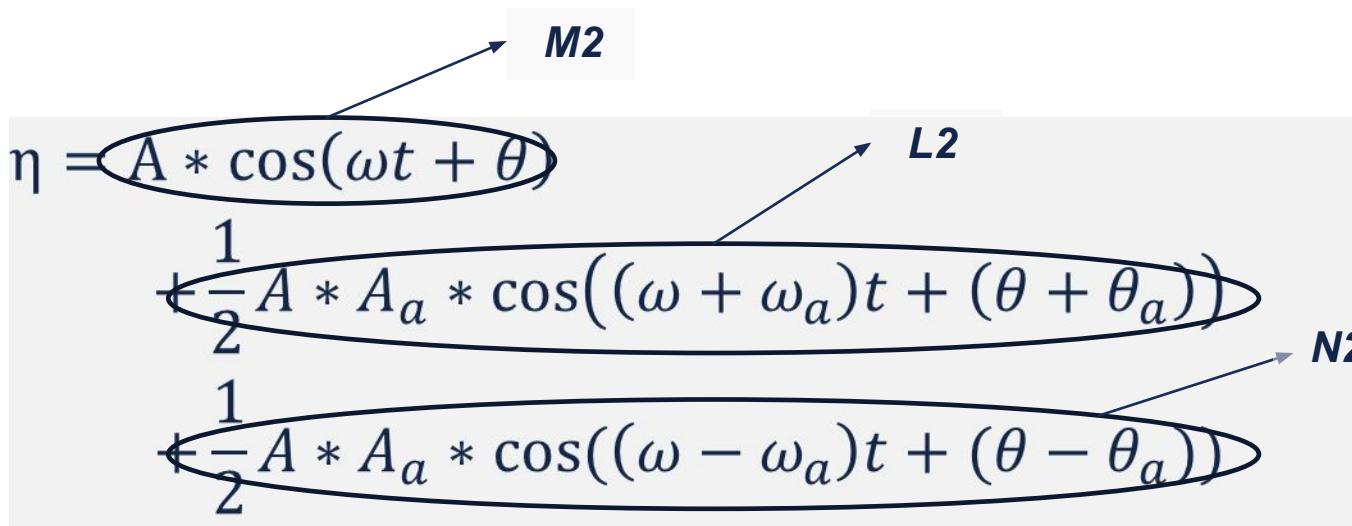
$$+ \frac{1}{2} A * A_a * \cos((\omega - \omega_a)t + (\theta - \theta_a))$$

If we consider that the moon's orbit is elliptical (anomalistic month 0.544°/hr)

$$M2 = 28,984 \text{ °/hr}$$

$$L2 = 28,984 \text{ °/hr} + 0,544 \text{ °/hr} = 29,528 \text{ °/hr}$$

$$N2 = 28,984 \text{ °/hr} - 0,544 \text{ °/hr} = 28,440 \text{ °/hr}$$

$$\eta = A * \cos(\omega t + \theta) + \frac{1}{2} A * A_a * \cos((\omega + \omega_a)t + (\theta + \theta_a)) + \frac{1}{2} A * A_a * \cos((\omega - \omega_a)t + (\theta - \theta_a))$$


We have assigned the subscript 2 to these new waves because they correspond to a semidiurnal wave periodicity. Their designations are as follows:

**M2 - Principal Lunar Semidiurnal Component.**

**N2 - Major Lunar Elliptical Semidiurnal Component.**

**L2 - Minor Lunar Elliptical Semidiurnal Component.**

If we introduce the lunar declination time... (TROPICAL MONTH 655.72 HOURS, BUT THE MOVEMENTS OF THE NORTHERN AND SOUTHERN DECLINATIONS ARE EQUAL, THEREFORE IT IS 327.86 HOURS)

$$360^\circ / 327.86 = 1.098^\circ/\text{hr.}$$

**This effect modifies the basic lunar wave M2.**  $M2 = 28.984^\circ/\text{hr}$

$$K1 = (28.984^\circ/\text{hr} + 1.098^\circ/\text{hr}) / 2 = 15.041^\circ/\text{hr}$$

$$O1 = (28.984^\circ/\text{hr} - 1.098^\circ/\text{hr}) / 2 = 13.943^\circ/\text{hr}$$

Following the previously explained nomenclature, these two new waves **K1** and **O1** have been assigned the suffix "**1**" because they will have a single cycle per day.

Similarly, we will deduce two other waves which we will designate as:

**K1 - Diurnal Lunisolar Declination Component** and

**O1 - Main Diurnal Lunar Declination Component.**



**DIURNAL  
INEQUALITY**

### Solar Tide:

In the case of solar tides, we will have similar results. The fundamental solar tide wave corresponds to **two tides in an exact solar day of 24 hours**.

This wave will be called **S2**

$$S2 = 360^\circ / 12\text{hr} = 30^\circ/\text{hr}$$

To compensate for the elliptical orbit, this **cycle between two Perihelions (Anomalistic Year)** has a period of **365.2596 days**.

Therefore, the speed of this cycle is:

$$360^\circ / (365.2596 \text{ days} \times 24 \text{ hr}) = 360^\circ / 8766.2304 = 0.041067^\circ/\text{hr}$$

$$S2 = 30^\circ/\text{hr}$$

$$R2 = 30^\circ/\text{hr} + 0.041067^\circ/\text{hr} = 30,041067^\circ/\text{hr}$$

$$T2 = 30^\circ/\text{hr} - 0.041067^\circ/\text{hr} = 29,958933^\circ/\text{hr}$$

**S2 - Main Semidiurnal Solar Component**

**R2 - Minor Elliptical Semidiurnal Solar Component**

**T2 - Major Elliptical Semidiurnal Solar Component**

Regarding the effects of **solar declination**, it will impact the solar tide wave, that is, the **S2** component.

The cycle we want to resolve is the one occurring between two maximum solar declinations. This cycle corresponds to the **Tropical Year** of **365.2422 days**, or equivalently, **8765.81 hours**.

As previously mentioned for the Moon, the maximum northern and southern declinations produce the same effects, so our cycle will have a period of  $8765.81/2 = 4382.905$  hours. Therefore, the angular velocity of this cycle will be:

$$360^\circ / 4382.905 \text{ hr} = \mathbf{0.08214^\circ/hr}$$

$$\mathbf{S2 = 30^\circ/hr}$$

$$\mathbf{K1 = (30^\circ/hr + 0.08214^\circ/hr)/2 = 15,041^\circ/hr}$$

$$\mathbf{P1 = (30^\circ/hr - 0.08214^\circ/hr)/2 = 14,959^\circ/hr}$$

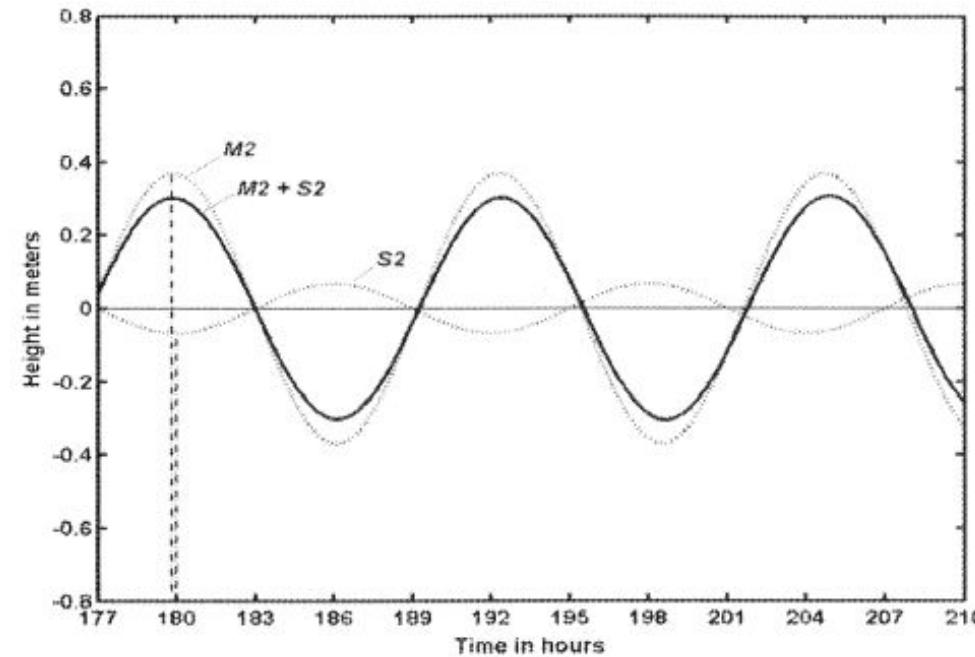
In this case, we observe that we obtain a wave again, the **K1**, which was previously obtained. Therefore, this wave **K1** is named:

**K1 - Diurnal Lunisolar Declination Component.**

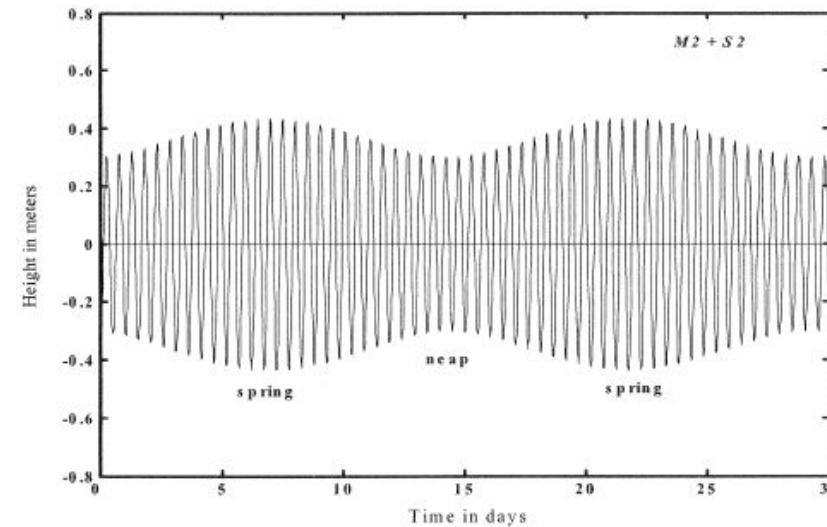
Similarly, the obtained wave **P1** is referred to as:

**P1 - Principal Diurnal Solar Declination Component.**

## Spring and neap tides

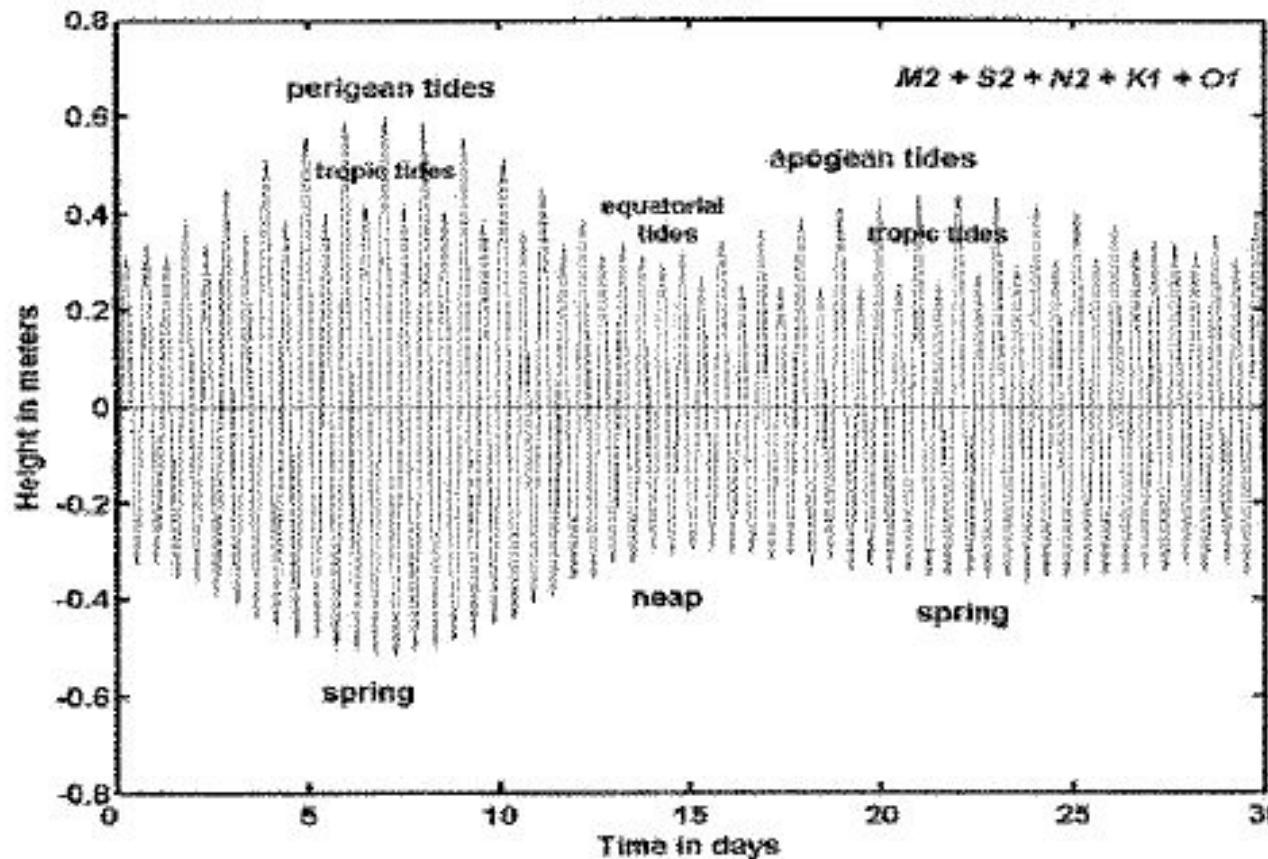


$M_2 + S_2$



$M_2 + S_2 + N_2 + K_1 + O_1$

Diurnal Inequality



## Long Period Components

In our study, we have added new diurnal and semidiurnal components to the basic M2 and S2 tides. To fully account for astronomical and climatic effects, it is necessary to include other waves referred to as **Long Period Components**.

The **Long Period Components** correspond to perturbations that do not produce diurnal or semidiurnal effects; instead, their influences exhibit longer periodicities such as biweekly, monthly, semiannual, annual, and so forth.

Decadal, biweekly, monthly, semiannual, and annual duration waves will not have solely an astronomical cause but will also encompass climatic factors. Sea level rise will fluctuate with the alternation of storms and anticyclones, wind climatology, seasonal changes, etc. These long-period waves will also include all this variability.

Thus, waves will appear with periods such as the **Mfm** of 9 days, **Mf** of 14 days, **Msf** of 15 days, **Mm** of 27 days, **MSm** of 31 days, **Ssa** of 6 months, and **Sa** of 1 year. These waves allow for the "*final adjustment*" of the variations corresponding to these periodicities for the synthetic tidal signal we are producing.

NOMBRE DE LA COMPONENTE	SÍMBOLO	VELOCIDAD	PERIODO	COEFTE.
Lunar Semidiurno Principal	M2	28.9841	12.42	1
Solar Semidiurno Principal	S2	30.00	12.00	0.466
Lunar Semidiurno Elíptico Mayor	N2	28.4397	12.66	0.192
Lunar-Solar Semidiurno	K2	30.0843	11.97	0.127
Solar Semidiurno Elíptico Mayor	T2	29.9589	12.01	0.027
Lunar Semidiurno Elíptico Menor	L2	29.5285	12.19	0.028
Solar Semidiurna Elíptico Menor	R2	30.0411	11.98	
Lunar-Solar Diurno Declinatoria	K1	15.0411	23.93	0.584
Lunar Diurno Principal	O1	13.9430	25.82	0.415
Solar Diurno Declinatoria Principal	P1	14.9589	24.07	0.194
Lunar Diurno Elíptico Mayor	Q1	13.3986	26.87	0.079
Lunar Diurno Elíptico Menor	M1	14.4967	24.84	0.033
	Mfm	1.6424	219.19	
Lunar Quincenal	Mf	1.098	327.9	0.172
Lunar Varacional Quincenal	Msf	1.0159	354.57	0.009
Lunar Mensual	Mm	0.544	661.3	0.091
	MSm	0.4715	763.52	
Solar Semianual	Ssa	0.082	4383	0.080
Solar Anual	Sa	0.041	8780.5	0.013

## OVERTIDES OR SUPERHARMONIC COMPONENTS

The basic astronomical components we have explained so far would allow us to represent the tidal wave in the open sea.

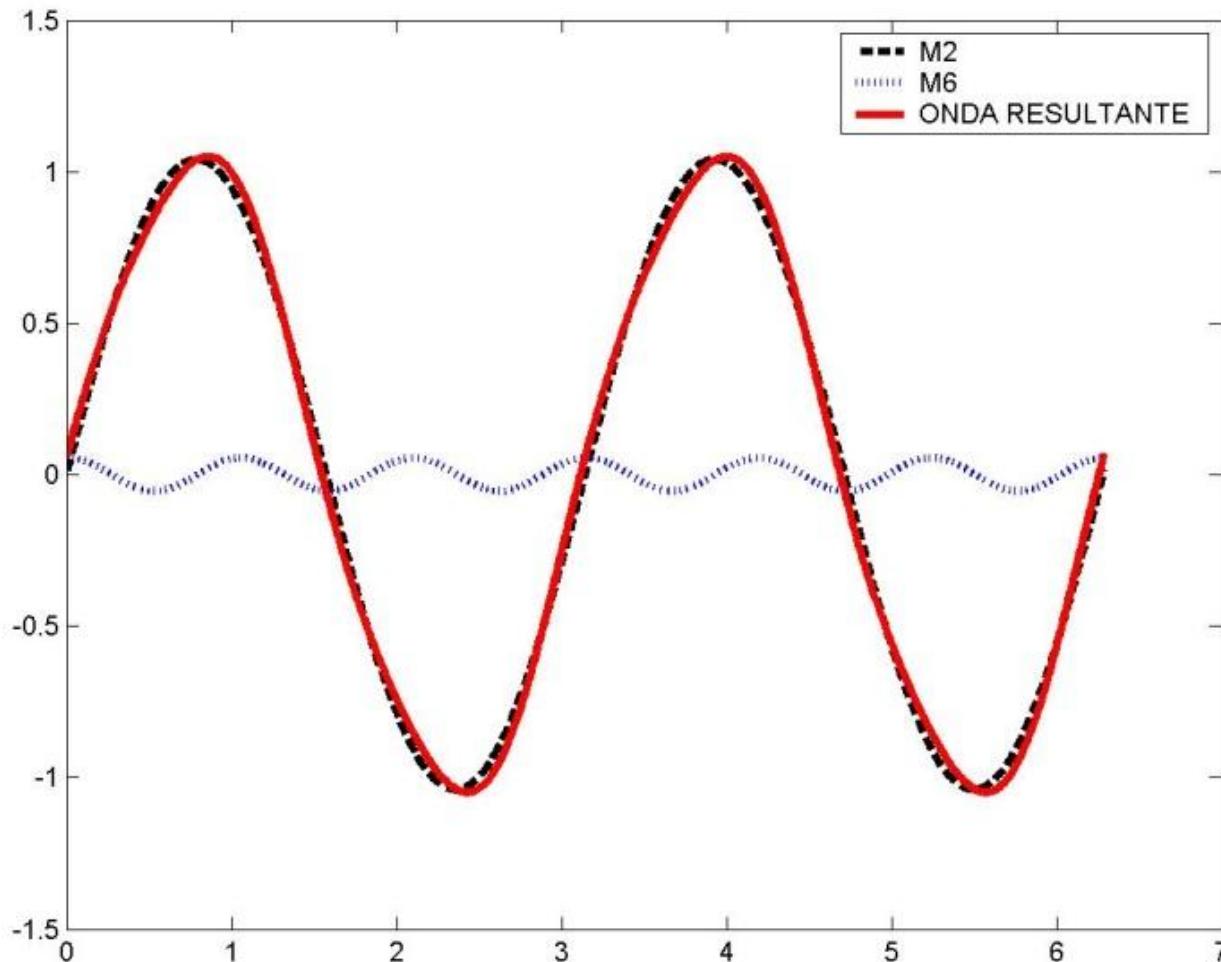
However, **long waves undergo a series of transformations when propagating in the shallow waters of coastal areas.**

Because of this, it is necessary to introduce elements that allow us to properly deform the tidal wave so that our synthetic tidal wave resembles the real tidal wave as closely as possible.

Therefore, to accurately deform tidal waves, it is necessary to add double, triple, and higher multiples of the frequencies of each of the basic waves initially calculated.

These waves with frequencies that are multiples of the basic wave frequencies will be referred to as **Overtides or Superharmonic Components.**

Thus, the Overtides include both the triple frequencies that represent bottom friction and the double frequencies that represent the difference in propagation speed between high tide and low tide.



**To reproduce the effect of friction in a wave, it is necessary to add another wave with triple the frequency.**

This means that for each harmonic component obtained so far, another component with triple the original frequency must be added. For example, if we have the waves M2, S2, etc., we should include in the list the waves M6, S6, and so on. In this way, we simulate the effects corresponding to bottom friction acting on these waves.

Another effect that must be considered is the difference in propagation speed between the crest (high tide) and the trough (low tide) of the tidal wave.

The propagation speed of a wave is proportional to the square root of the depth.

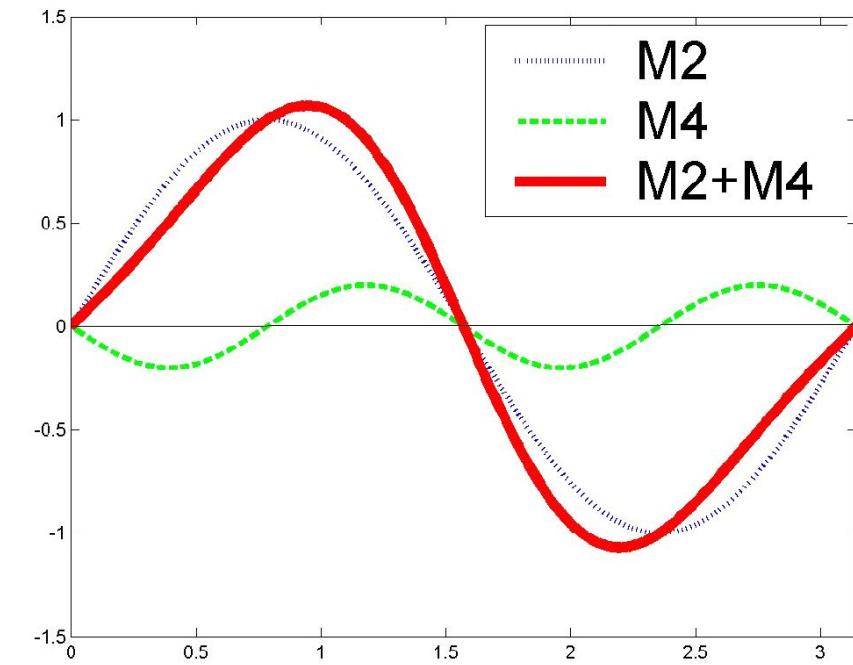
In the case of the trough of the wave (low tide), the depth is lower, and the propagation speed will be less than that corresponding to the crest (high tide).

$$c = \sqrt{gh}$$

$$c = \sqrt{g(h - A)}$$

$$c = \sqrt{g(h + A)}$$

As we have stated so far, all effects should be attempted to be reproduced by adding other waves to our calculation. As we will see graphically, the way to reproduce this deformation is by adding waves of double frequency, quadruple frequency, etc. In this way, **for each wave M2, S2, etc., its corresponding double frequency wave M4, S4, M8, S8, etc., will appear.**



ALGUNOS EJEMPLOS DE ONDAS COMPUESTAS	
SIMBOLO	PROCEDENCIA
MNS2	M2+N2-S2
2MS2	2M2-S2
2SM2	2S2-M2
MK3	M2+K1
2MK3	2M2-K1
SK3	S2+K1
SO3	S2+O1
MS4	M2+S2
MN4	M2+N2
MK4	M2+K2
2MS6	3M2+S2
2MN6	2M2+N2
2SM6	2S2+M2
MSN6	M2+S2+N2
3MS8	3M2+S2
2MSN8	2M2+S2+N2

## COMPOUND TIDES

Another effect to consider is the interactions that the waves we have observed so far will undergo upon reaching shallow coastal waters and combining with one another.

Wave interaction is not a linear process. As a result of the combination of multiple waves, we obtain new waves with frequencies corresponding to nonlinear combinations of the original frequencies.

For example, the combination of the M2 and S2 waves can give rise to the appearance of waves such as 2MS2, 2SM2, MS4, 2MS6, 2MS8, etc.

These new waves will have frequencies that are nonlinear combinations of the frequencies of M2 and S2.

Since combinations can occur not only between two waves but also among three, four, or more waves, the list of components will increase substantially.

Only observation and analysis will allow identifying which components are present at a given location.

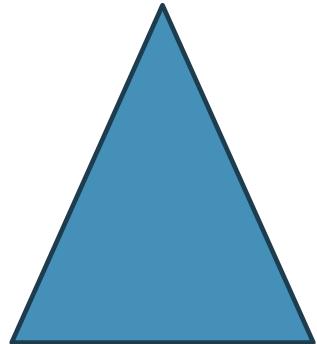
<https://tidesandcurrents.noaa.gov/harcon.html?id=9410170>

## B) TIDAL REGIME

THE ORDER IS PEACE. MARIE KONDO

## HOW CAN YOU MAKE A CLASSIFICATION?

BY FORM



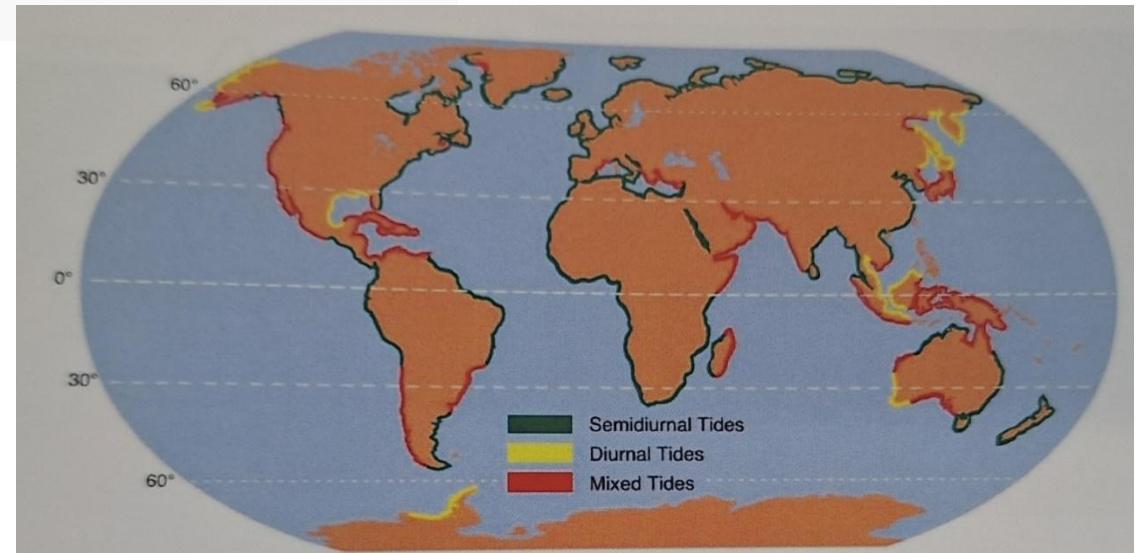
BY SIZE



## FORM FACTOR

$$F = \frac{A_{K1} + A_{O1}}{A_{M2} + A_{S2}}$$

$$F = \frac{K_1 + O_1}{M_2 + S_2} = \frac{\text{diurnal harmonic amplitude}}{\text{semidiurnal harmonic amplitud}}$$



$F =$

0-0,25

0,25-1  
,5

1,5-3

Mayor  
que 3

Semidiurnas  
Puro

Mixto -  
Semidiurno

Mixto-Diu  
rno

Diurnas  
Puro

The previous classification has been made based on the periods of the components involved in tide generation but does not specify anything regarding their magnitude.

Using the **ranges** of spring tides, the following classification is commonly applied:

### Type:

Micromareal < **2** m

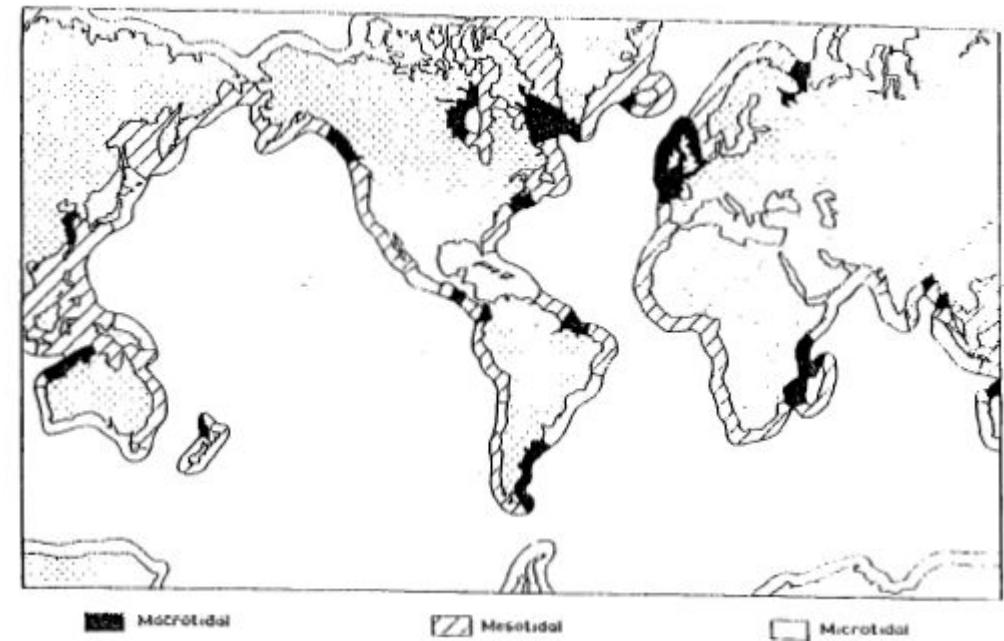
Mesomareal 2 - 4 m

Macromareal > **4** m

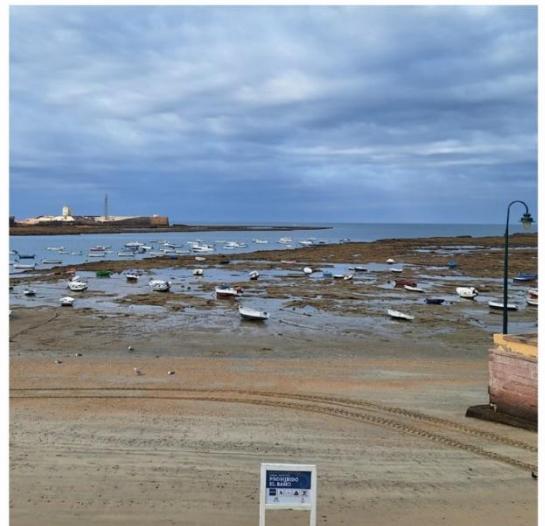
### **King tides**

While not a scientific term, 'king tide' is widely used to describe any remarkably high tide.

The king tides occur at new and full moon when the Earth, Moon and Sun are aligned at perigee and perihelion, resulting in the largest tidal range seen over the course of a year. So, tides are enhanced when the Earth is closest to the Sun around January 2 of each year. They are reduced when it is furthest from the Sun, around July 2.









**TIDAL BORES**



## C) AMPHIDROME SYSTEM

LET`S SEE FARAWAY OUR PORTS AND BEACHES.

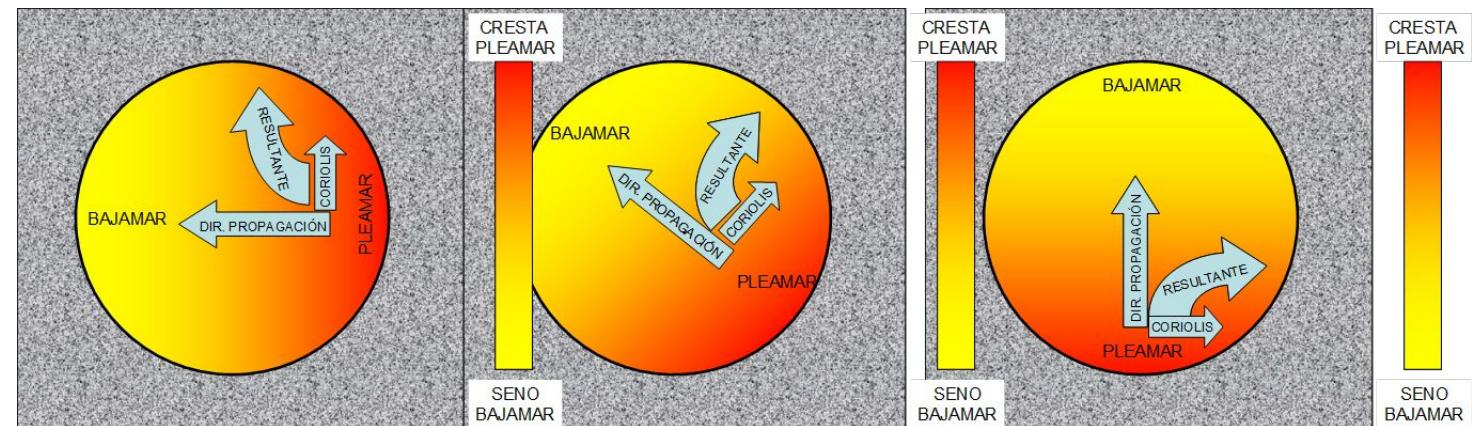
## THREE-DIMENSIONAL PROPAGATION - AMPHIDROMIC SYSTEMS

Up to this point, we have considered wave propagation in two dimensions, that is, we have examined how waves propagate longitudinally. However, real waves are three-dimensional, and we must take other aspects into account.

To begin with, the propagation of long waves such as tides occurs on a planetary scale. This scale implies that the Coriolis effect must be considered.

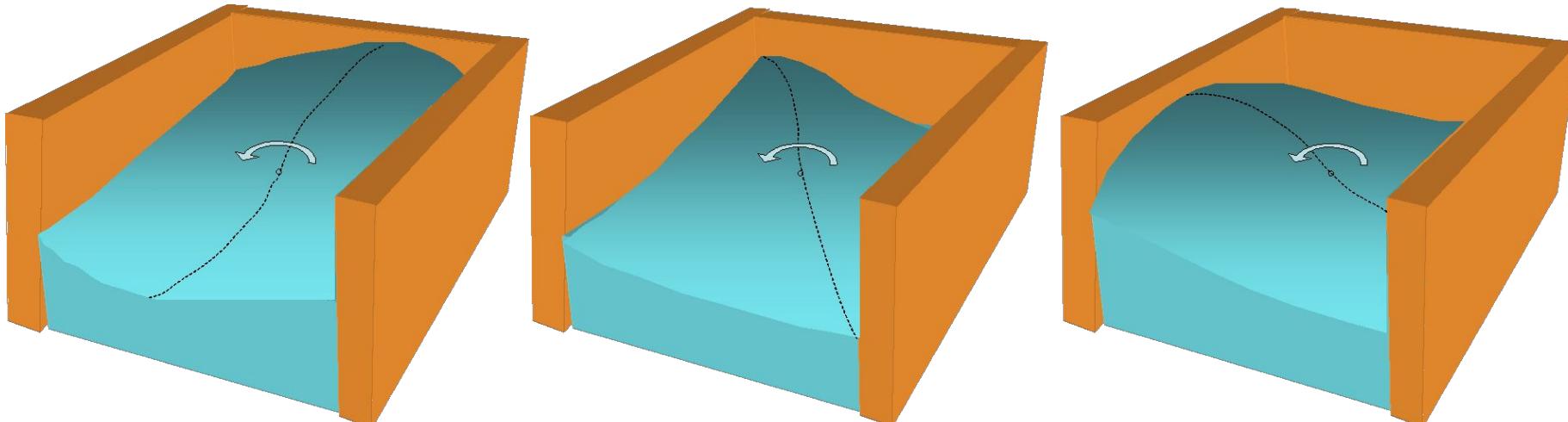
**The Coriolis effect causes large-scale movements in the Northern Hemisphere to deflect to the right (to the left in the Southern Hemisphere).**

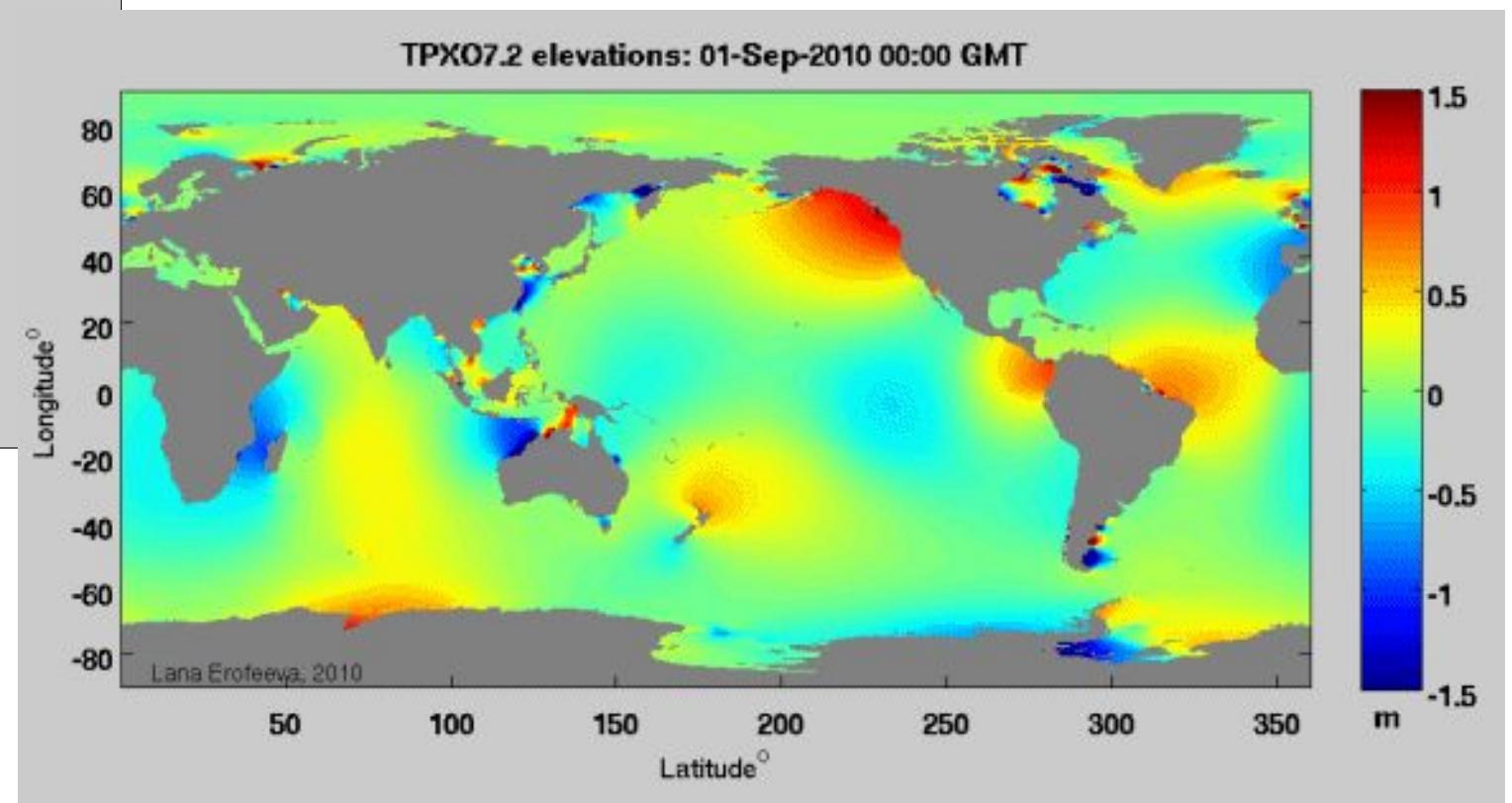
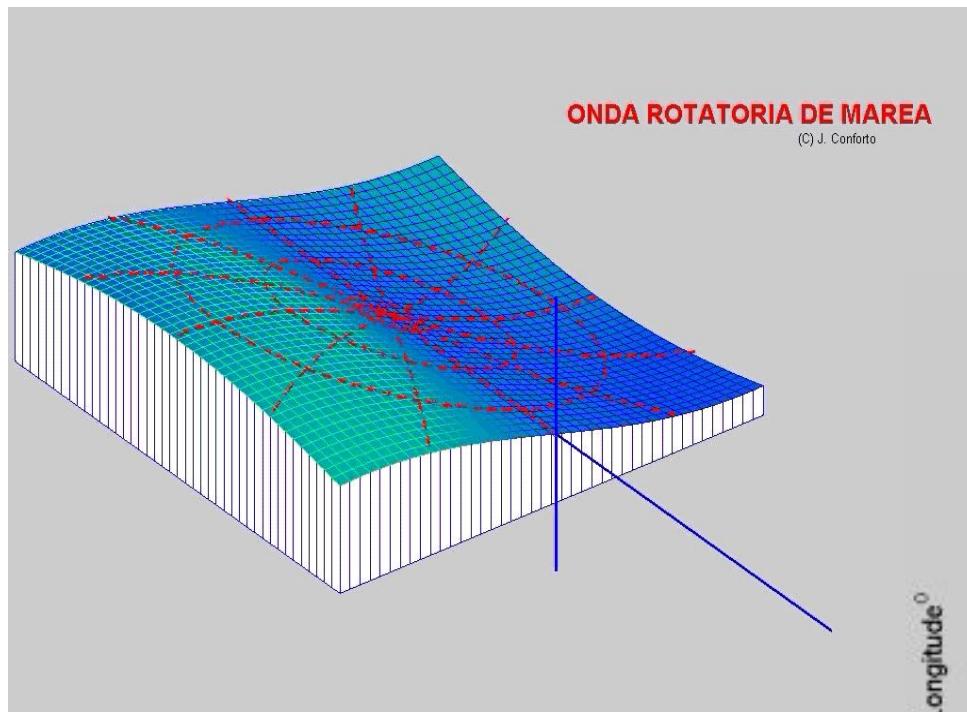
The Coriolis force is responsible for causing all geophysical movements on a planetary scale to tend to describe rotations. Because the Coriolis force acts to the right in the Northern Hemisphere, any large-scale movement will experience a continuous turn to the right. This is the cause of cyclonic and anticyclonic rotations in the atmosphere and also turns large ocean current systems into large circular gyres.



We can visualize this phenomenon by taking a plate filled with water and causing it to oscillate. A wave will form that travels along the perimeter of the plate, exhibiting greater amplitude at the plate's edge. This is precisely what we refer to as an Amphidromic system.

An Amphidromic System is therefore defined as one in which the crest of a wave propagates around an **Amphidromic Point** characterized by zero amplitude at its center, with amplitude increasing progressively towards the periphery.



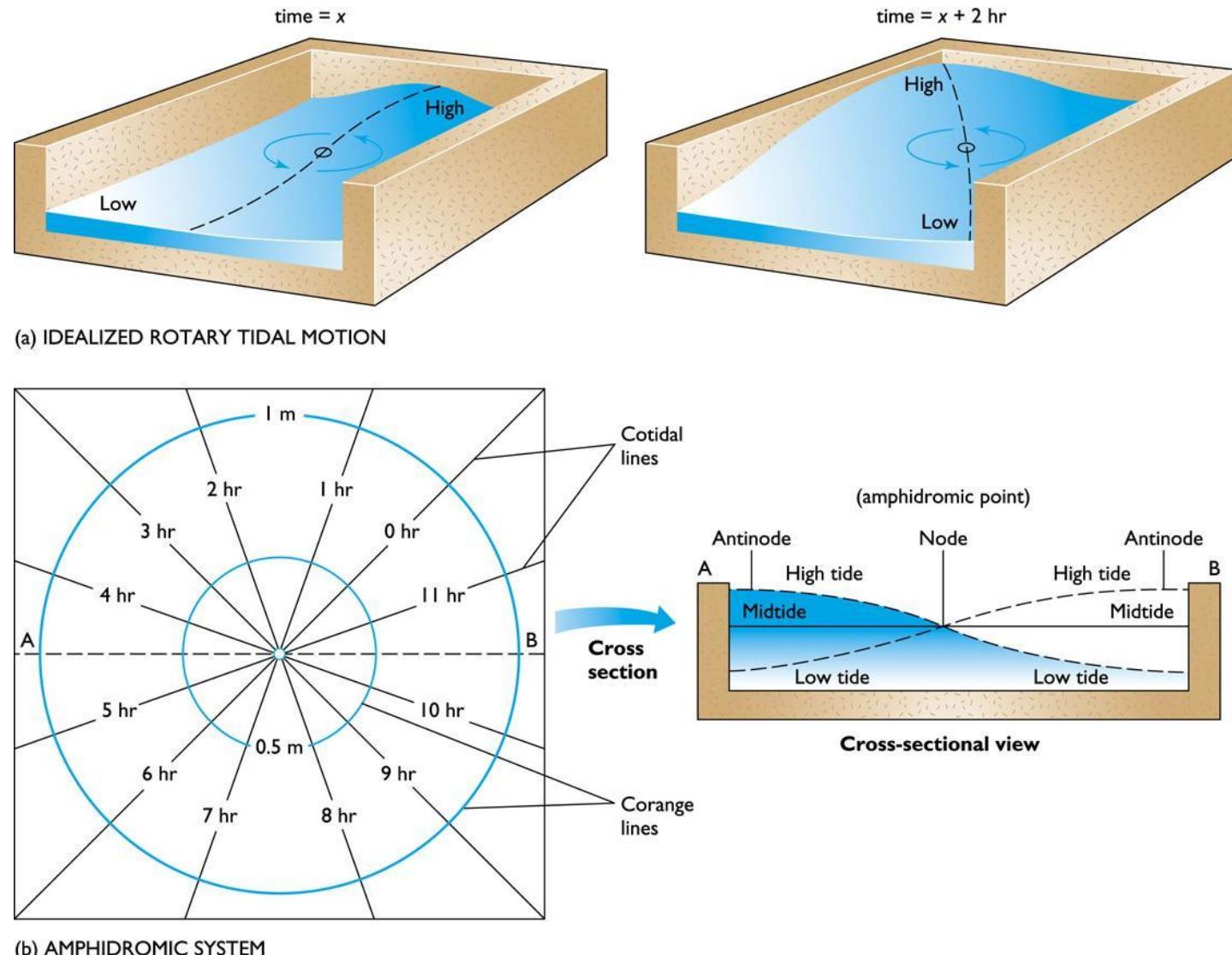


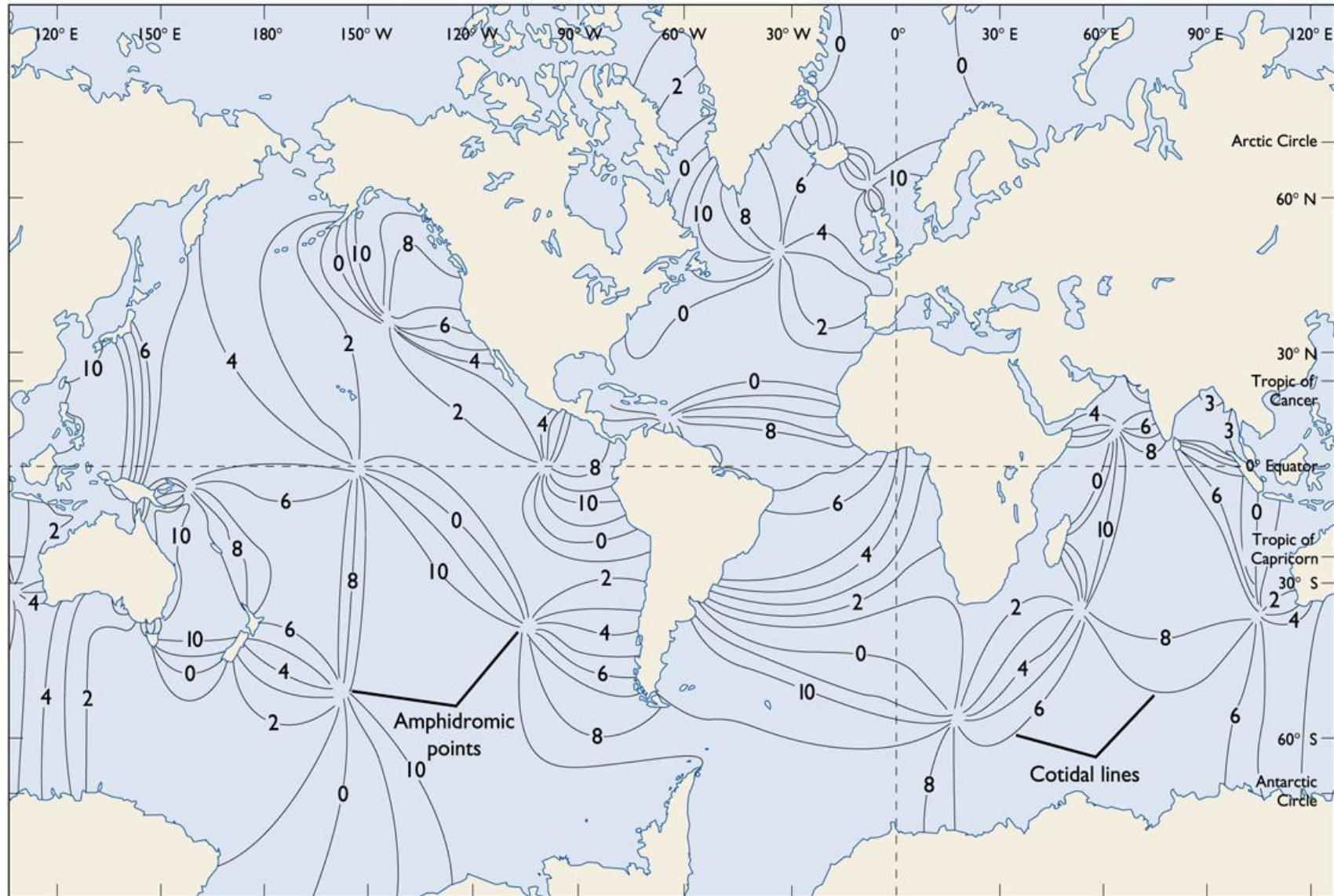
## D) COTIDAL AND CORANGES MAPS

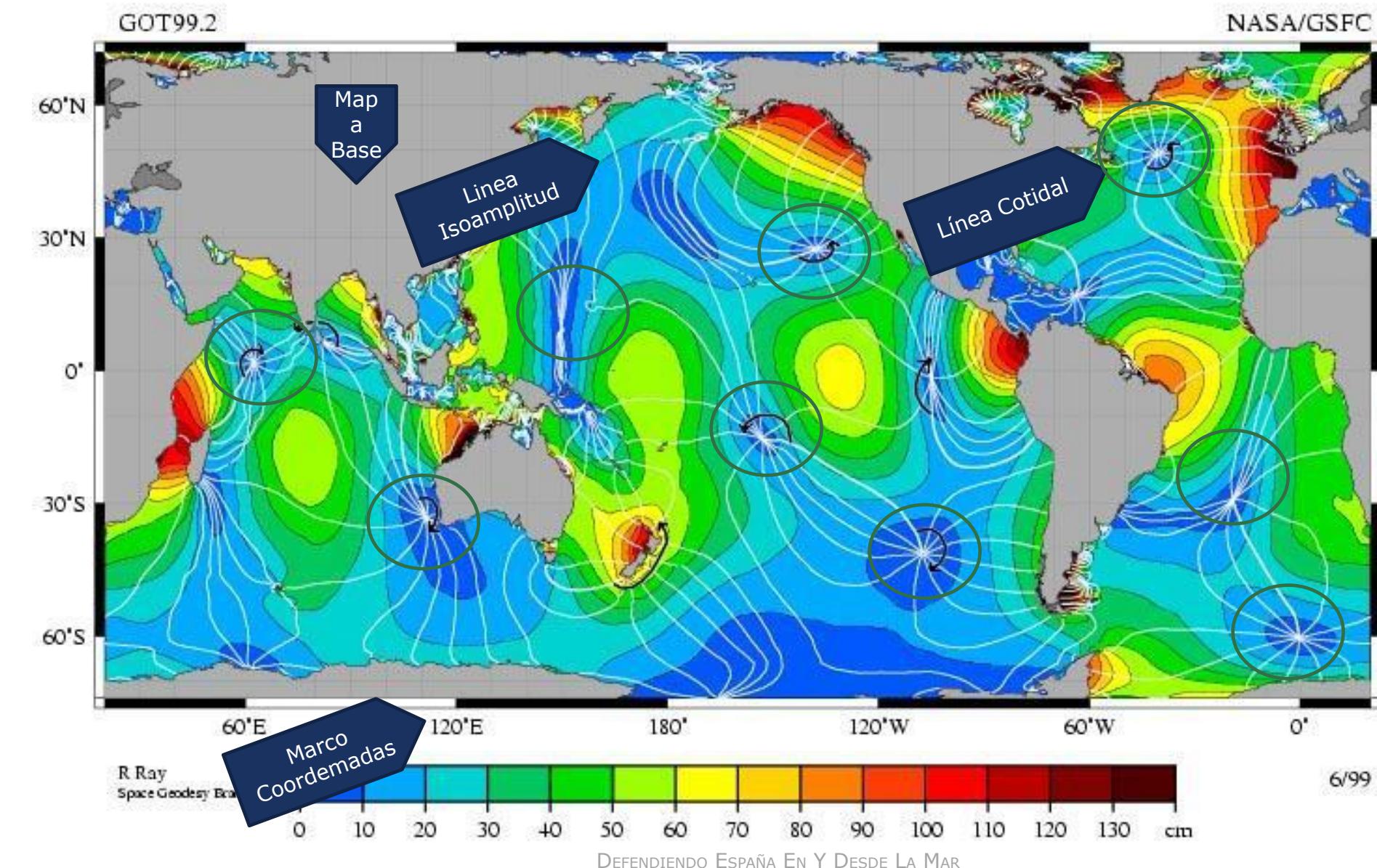
KNOW THE PAST TO MANAGE THE PRESENT.

A rotary wave is part of an Amphidromic system (rotating standing wave) in which the wave propagates around a node (where there is no vertical displacement) with the antinode (maximum vertical displacement) rotating along the coastline contour.

The cotidal lines connect points on the rotary wave that experience high tides simultaneously. Cotidal lines are not evenly spaced because the propagation speed of the tidal wave depends on the depth.



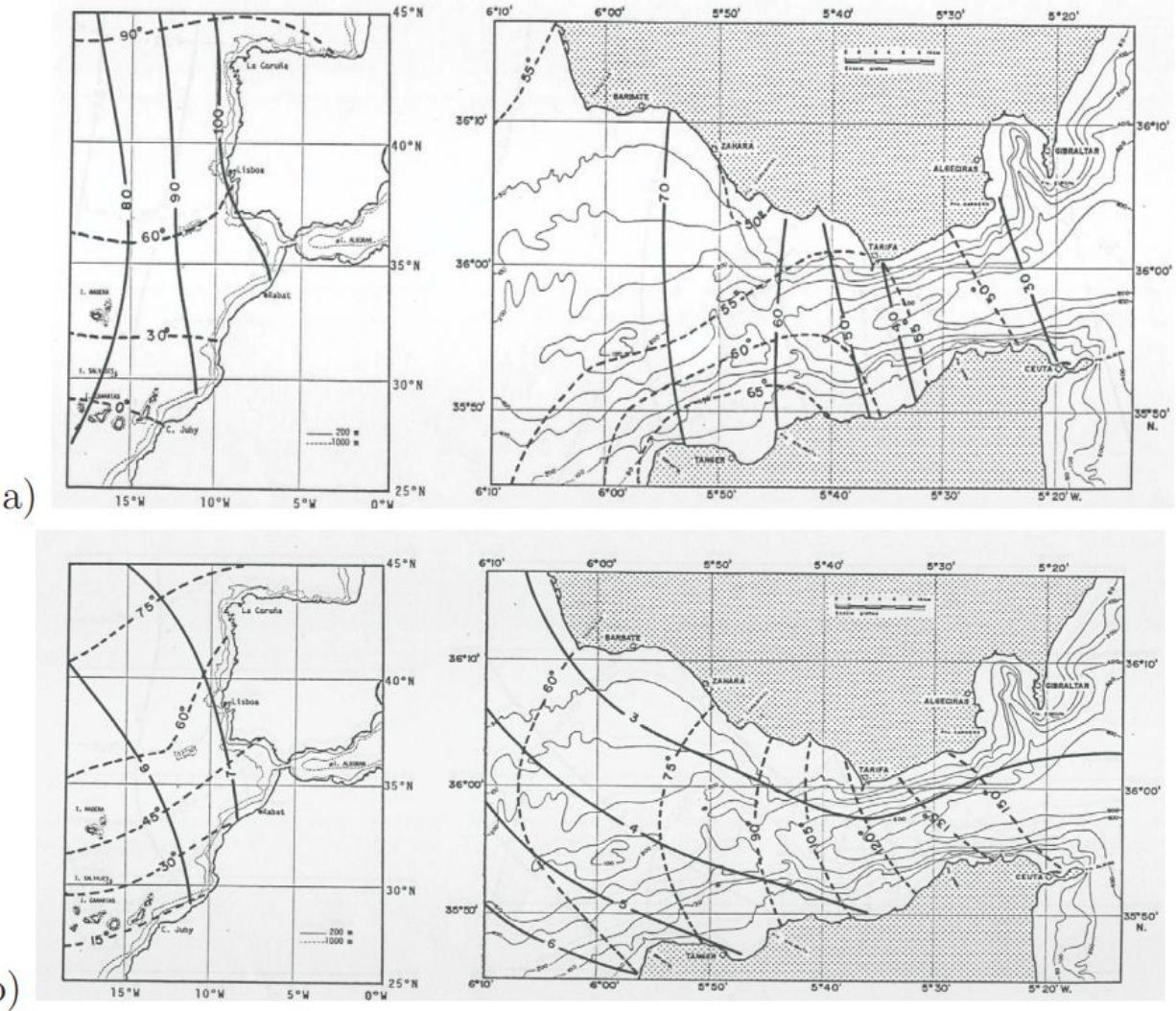




Cotidal or Corange Chart: it is the graphical representation of tides that indicates the times and heights of tides at different locations along a coastline.

Cotidal Line: it is a graphical representation of points on Earth where the water level rises and falls **simultaneously** due to the tide. These are lines of equal lunital interval.

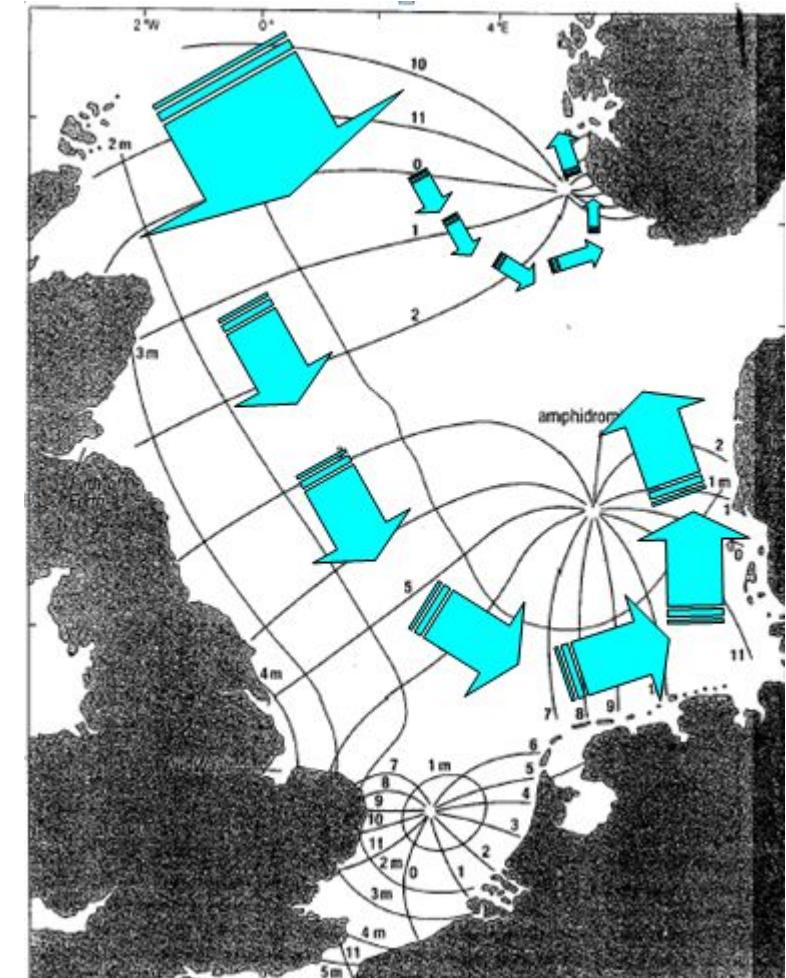
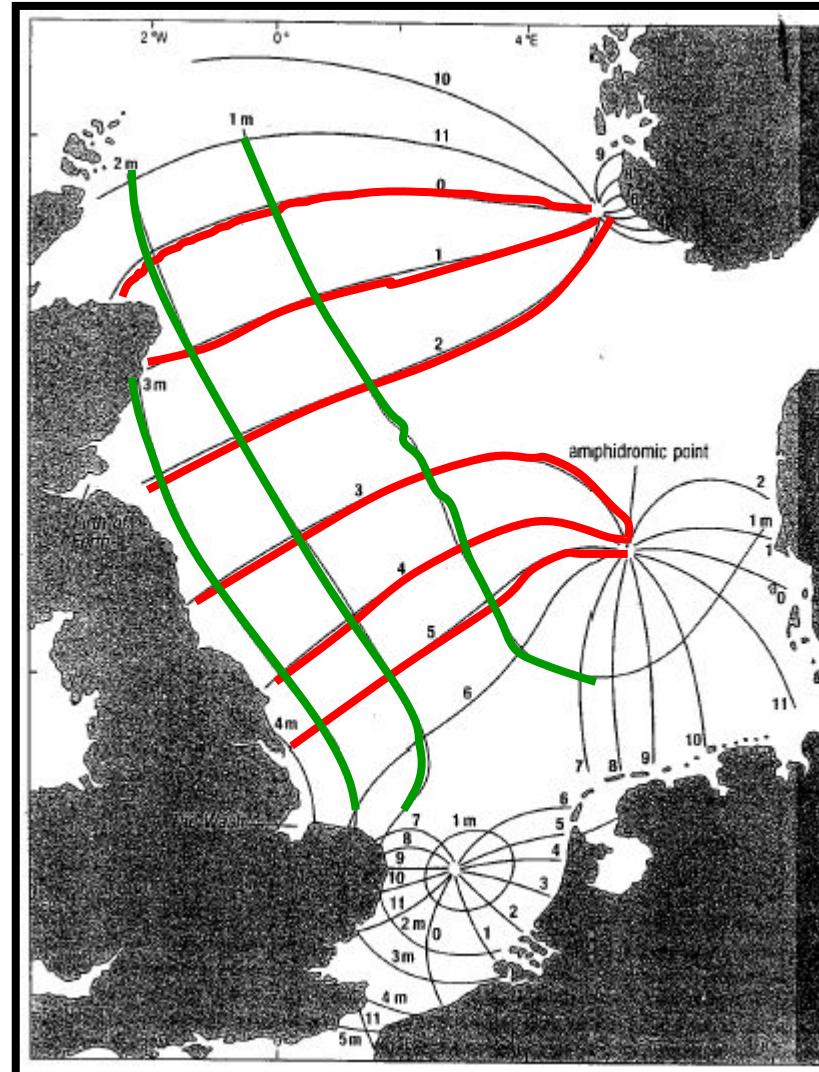
Co-Range Line: it is a graphical representation of the points on Earth where the water level rises and falls within the same range of heights due to the tide.



The amphidromic system corresponding to the  $M_2$  component in the North Sea.

In each Amphidromic System, the locus of points where the tide is at the same phase is usually defined, as well as the corresponding points of equal tidal range. These lines are approximately perpendicular to each other.

It should be noted that three amphidromic systems appear in the North Sea.

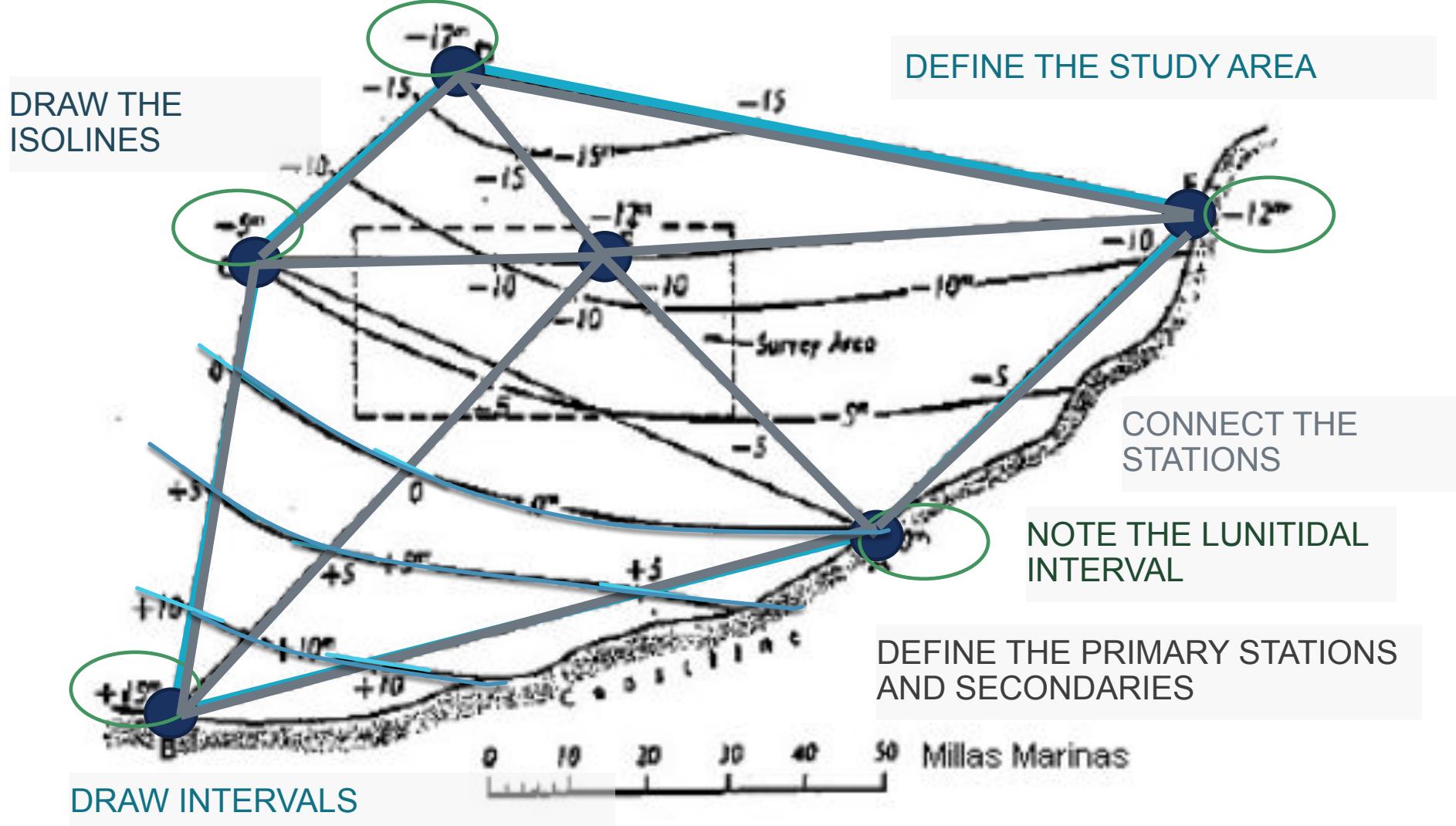


### Construction of a Cotidal Chart.

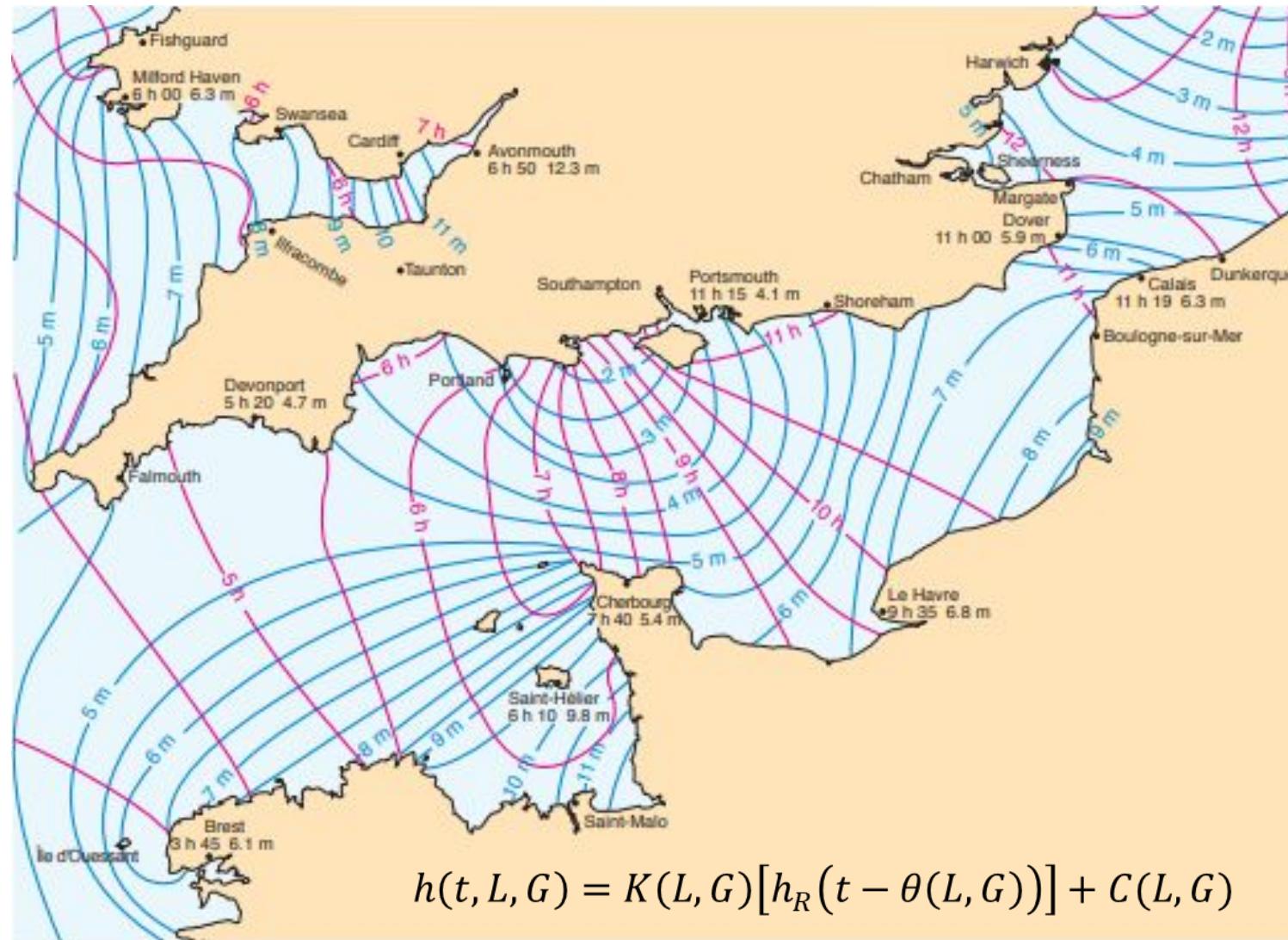
- Define the study area, which must contain within it an equilateral triangle or a quadrilateral formed by the tide gauge stations.
- Identify the reference station and all secondary, tertiary, and short-term tide gauge stations within the study area.
- For each station, record the arrival times of high tide and low tide (some charts require these two parameters separately, with one Cotidal chart for high tide and another for low tide).
- Adjacent and opposite stations are connected with straight lines.
- Periodic intervals are interpolated along each line. The time segments will depend on the tidal amplitude and the desired precision for the reductions (usually 10 minutes).
- The interval marks are connected with a smooth curve.
- When two interpolated points conflict, preference is given to the mark on the shorter line and to the marks on the lines that the curves intersect closer to perpendicularity.

Isoamplitude charts are constructed similarly but based on amplitude marks every 10 cm.

## CONSTRUCCION DE UNA CARTA COTIDAL CON CURVAS DE TIEMPO



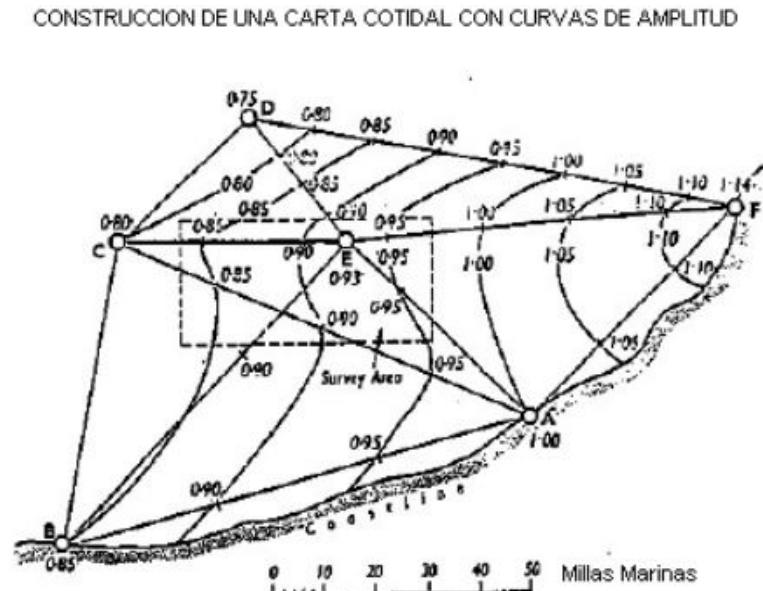
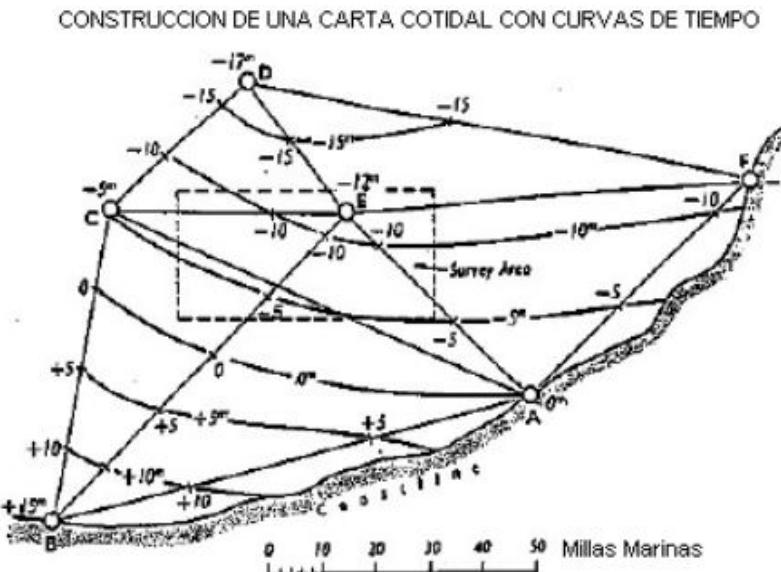
The **reduction of soundings** from cotidal charts.  
The figure shows a chart that can be used for the correction of soundings in deep waters (offshore) with a grid of equal high tide range lines (blue) and phase lines (pink) in the channel between England and France.



WATER LEVEL AND **TIDE ZONING** IS A TOOL USED TO EXTRAPOLATE AND INTERPOLATE TIDE OR WATER LEVEL VARIATIONS FROM THE NEAREST GAUGE STATION TO THE SURVEY AREA.

Zoning is the practice of dividing the survey area into discrete zones with uniform tidal characteristics.

Zoning Maps: These maps are created by overlaying the Cotidal map onto the isoamplitude map. They allow the selection of regions to apply amplitude and time corrections to the reference station for height and time.

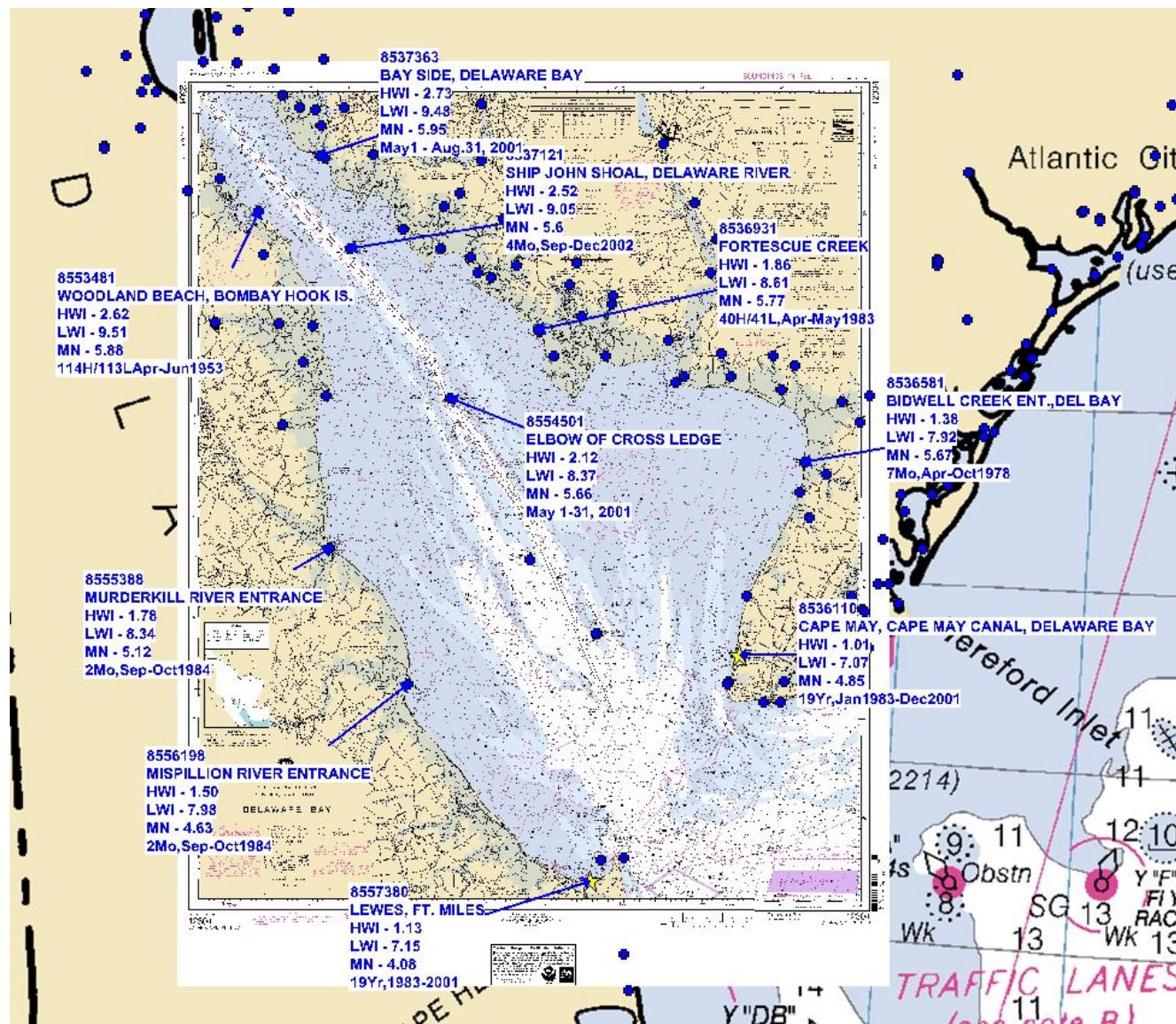


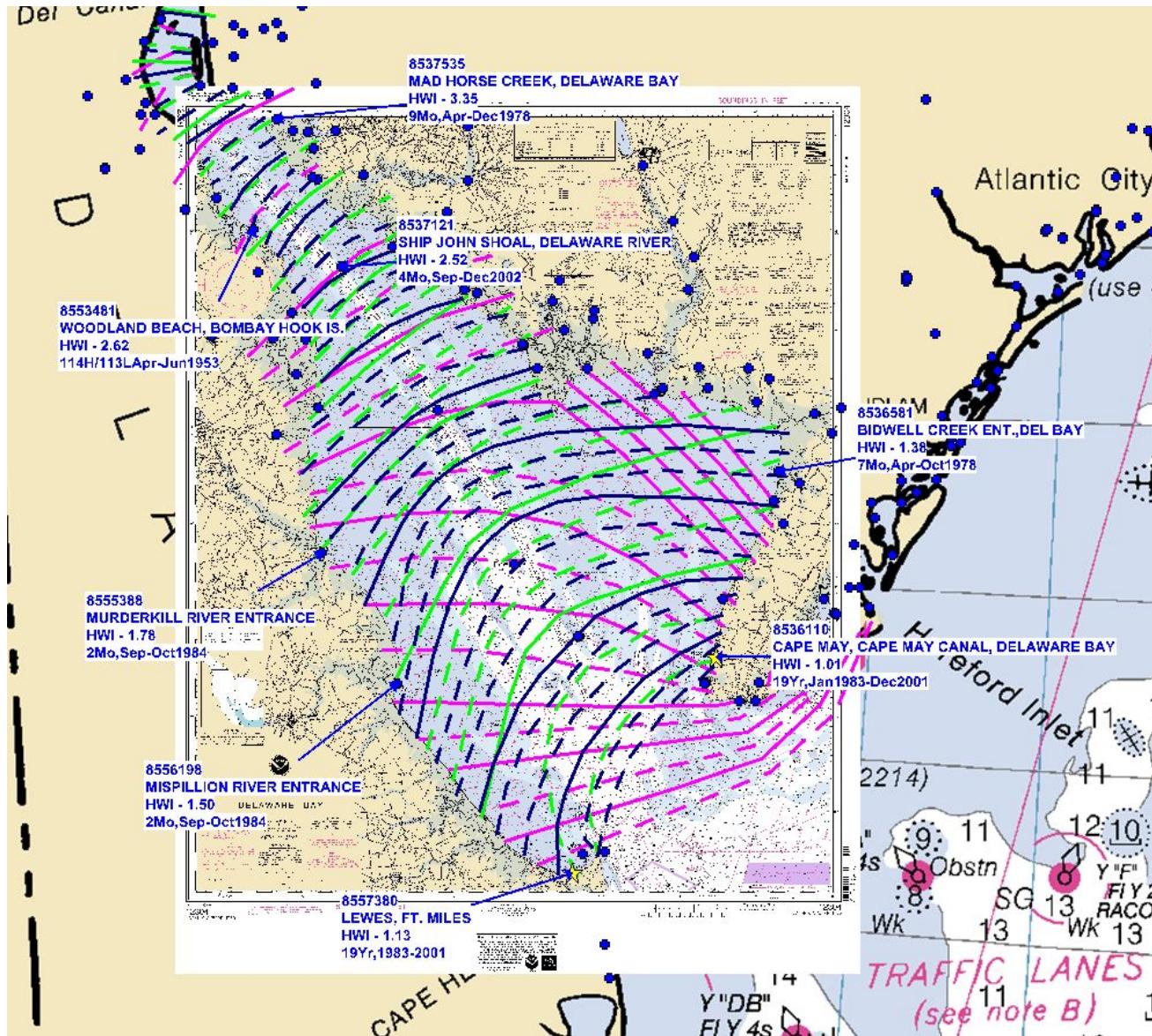
## Construction of the **final tide zoning schemes**.

The NOS currently employs the “discrete tide zoning” method for operations, where the survey areas are divided into a cell scheme in which the edges share common tidal characteristics. The minimum requirement for each new cell is a change of 0.06 m in average tide amplitude and a 0.3-hour delay in the tide curve.

	<b>Amplitud</b>	<b>Tiempo</b>
C-13 - IHO	0.06 m – 0.196 ft	0.3 h
USACE C-5	0.2 ft – 0.061 m	0.2 h

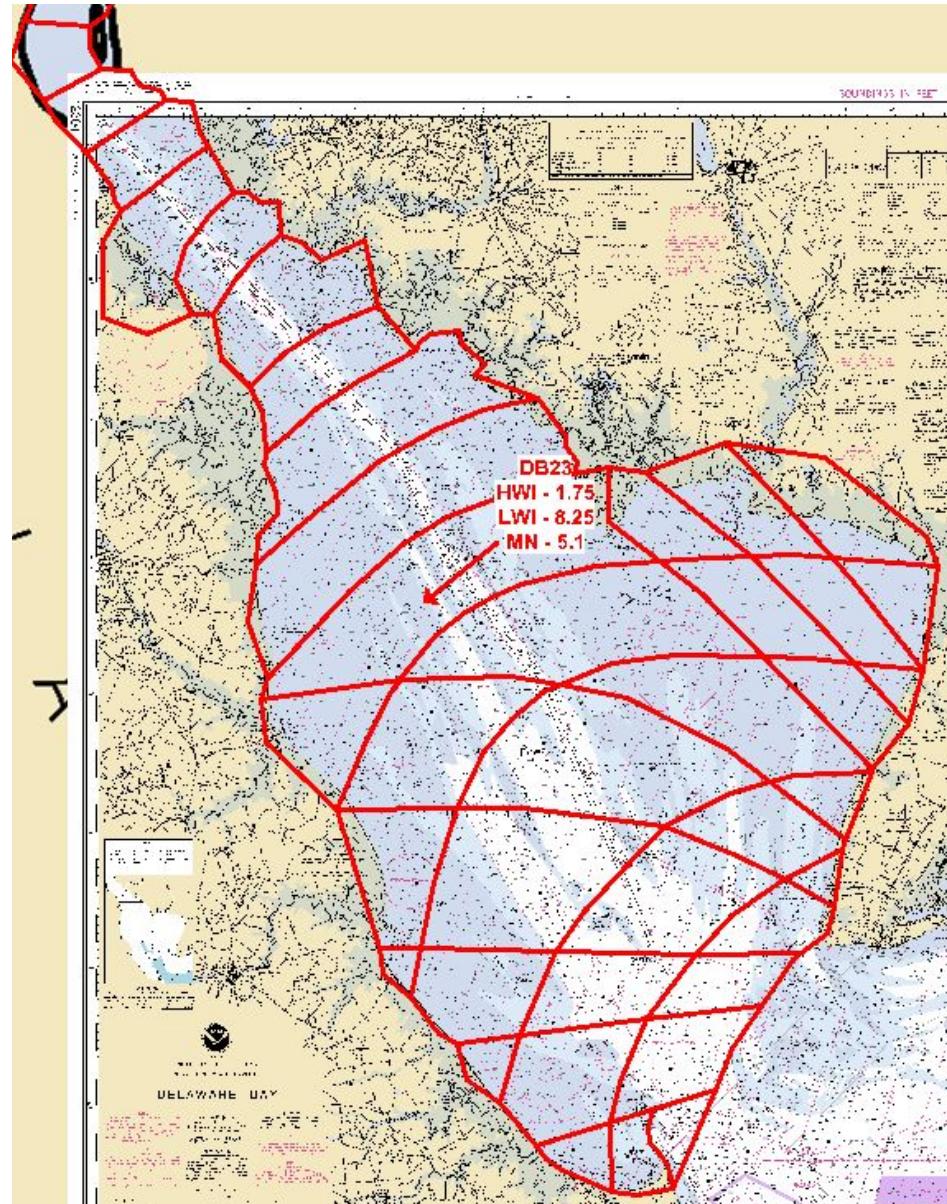
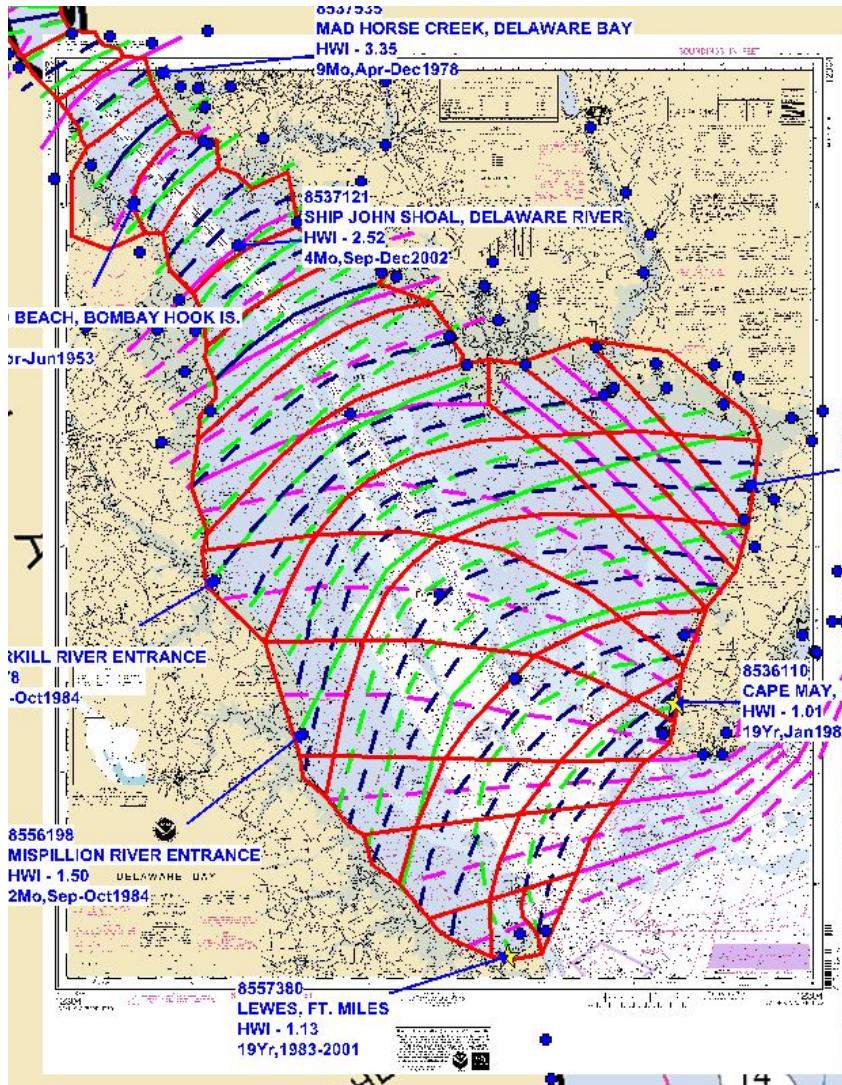
These features can be modified depending on the type of survey, location, or tidal characteristics.





A discrete tidal zone is to represent NO MORE than:

- 0.2 ft – range change
- 0.3 hours – phase change



Correction parameters are calculated based on the characteristic tide values and the values from the reference station.

HWI (zone) – HWI (control) = High Tide Corrector (HTC) – **High Tide Phase Corrector**

LWI (zone) – LWI (control) = Low Tide Corrector (LTC) – **Low Tide Phase Corrector**

MN (zone) / MN (control) = Range Ratio (RR) – **Amplitude Corrector**

## Example:

## Zone DB23:

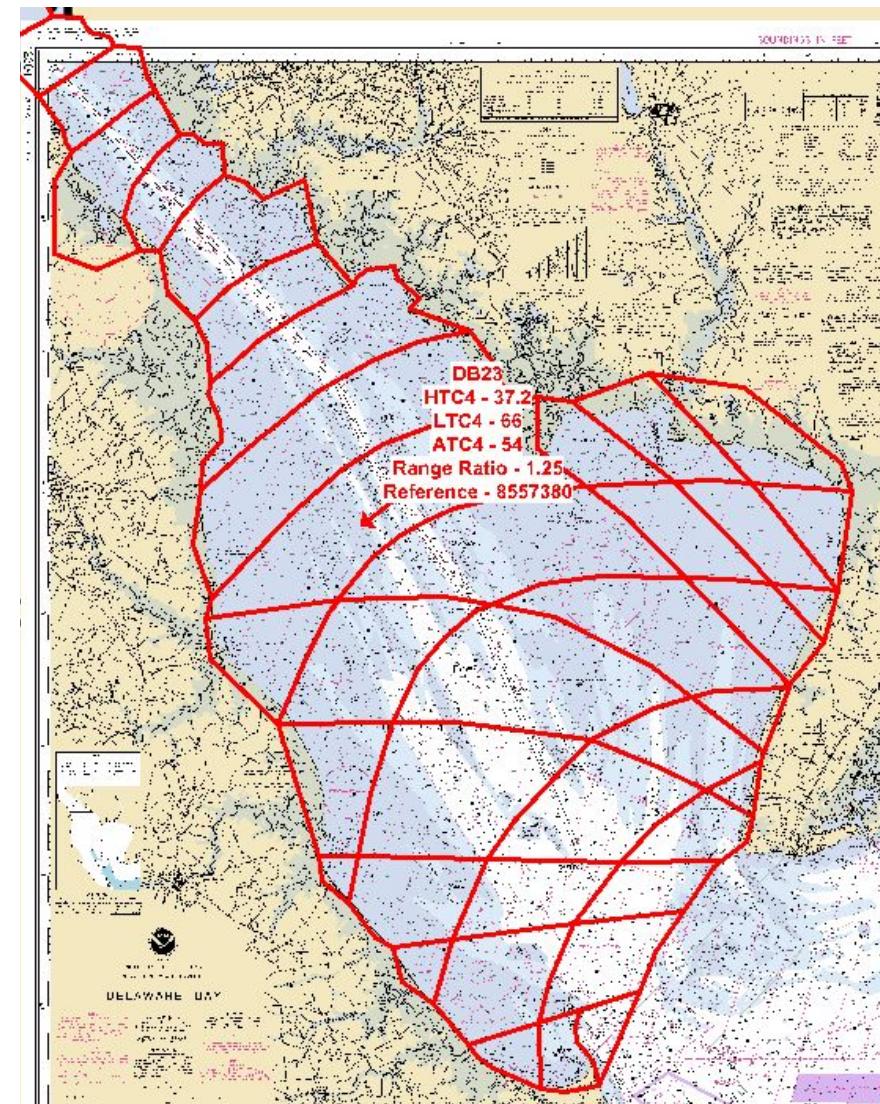
HWI – 1.75, LWI – 8.25, MN – 5.1

## Control (Lewes):

HWI – 1.13, LWI – 7.15, MN – 4.08

## Correction parameters:

High tide phase corrector: +36 min, Low tide phase corrector: +66 min,  
Mean phase corrector: +48 min, Amplitude correction factor: 1.25



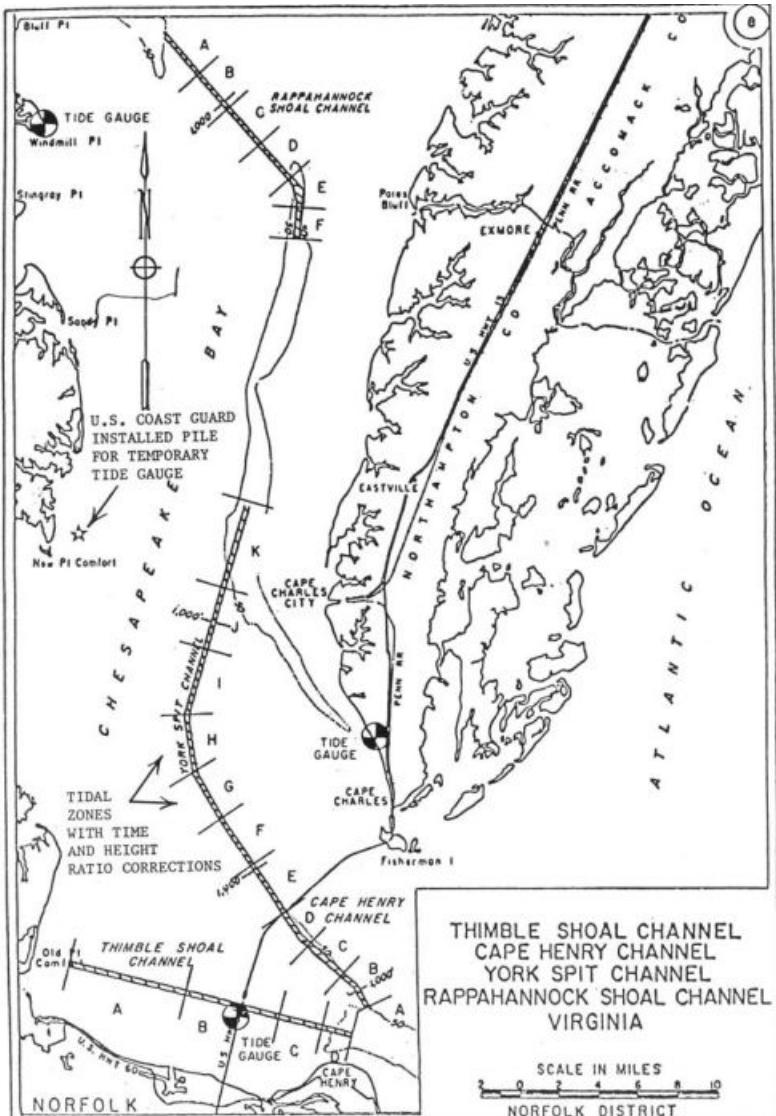


Figure 5-21. Chesapeake Bay tidal zones

Table 5-3. Cape Henry Channel Tidal Zoning  
 Tide Correction Factors Based on Chesapeake Bay Bridge Tunnel Gage  
 10 July 1991

Zones	Station to Station	Time of Occurrence	Height Ratio
A	-20+00 to +20+00	-23 min high water -17 min mean -11 min low water	x 1.21
B	+21+01 to +133+00	-17 min high water -13 min mean -09 min low water	x 1.15
C	+133+01 to +239+73	-11 min high water -9.5 min mean -08 min low water	x 1.08
D	+239+74 to +332+86	Direct on CBBT gage " " "	N/A

Example of tidal zoning correction for tidal zone B:

Channel Station 100+00 Time 1000 EST

Tide stage occurs 13 minutes (on average) later at CBBT tide gage ... i.e., 1013 EST

Gage reading at 1013 EST:  
height ratio: +2.00 ft above MLLW  
x1.15

Gage reading corrected for  
time and height to be applies  
to the 1000 EST sounding  
+ 2.30 ft above MLLW

On the left, the zoning of the Chesapeake Bay.  
 Above, the list of corrections from the tide gauge station.





IHM