IHM ARMADA

BARBADOS WORKSHOP

TIDAL THEORY

SILVIA COSTA

IHM

CONTENS

A. TIDE AS A LONG WAVE
B. PRINCIPLES OF THE THEORY OF THE EQUILIBRIUM
C. THE EARTH AND THE MOON
D. THE SUN
E. EARTH- MOON – SUN

A) TIDE AS A LONG WAVE

BUT WE´LL EXPLAIN IT AS A SHORT STORY

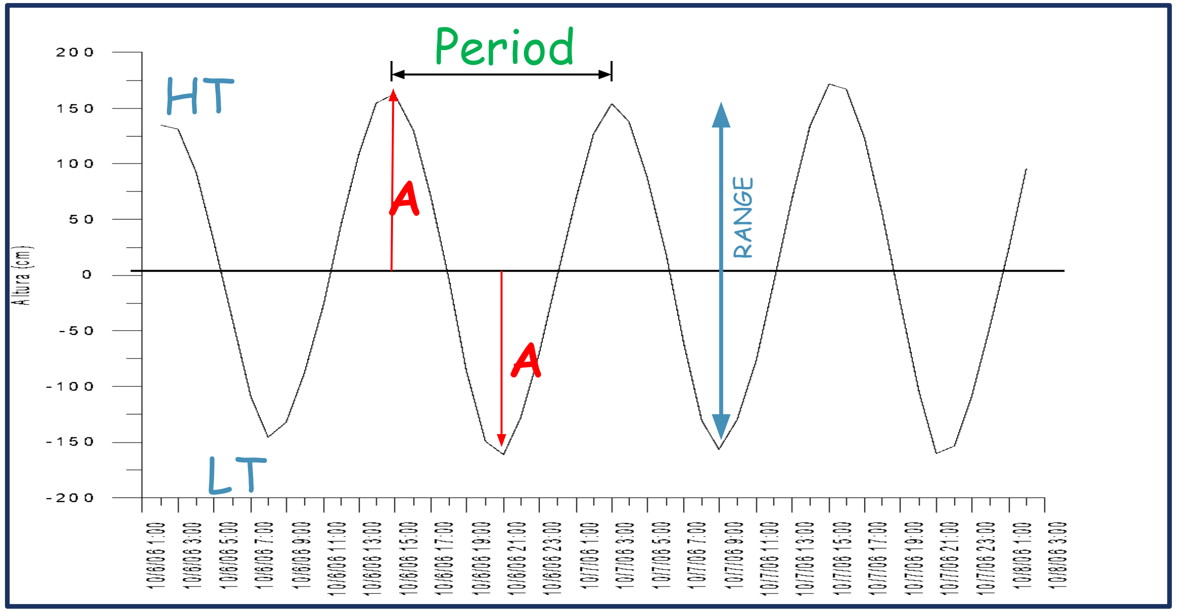
SIT WAIT AND SEE

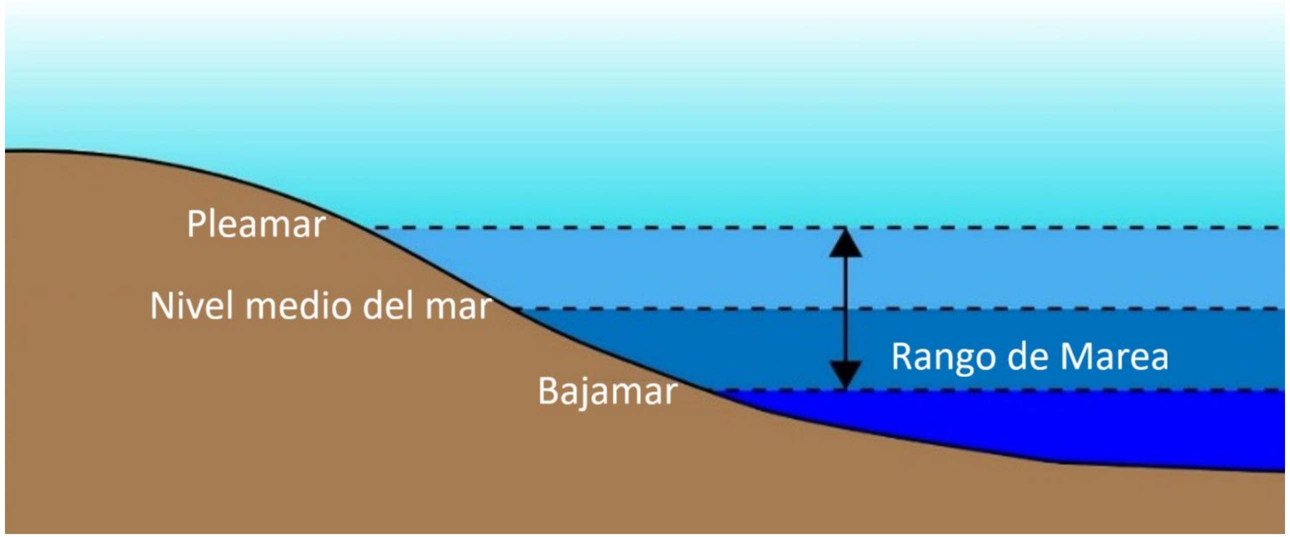
THE TIDE AS A WAVE

WAVE IS CARACTERIZASE BY

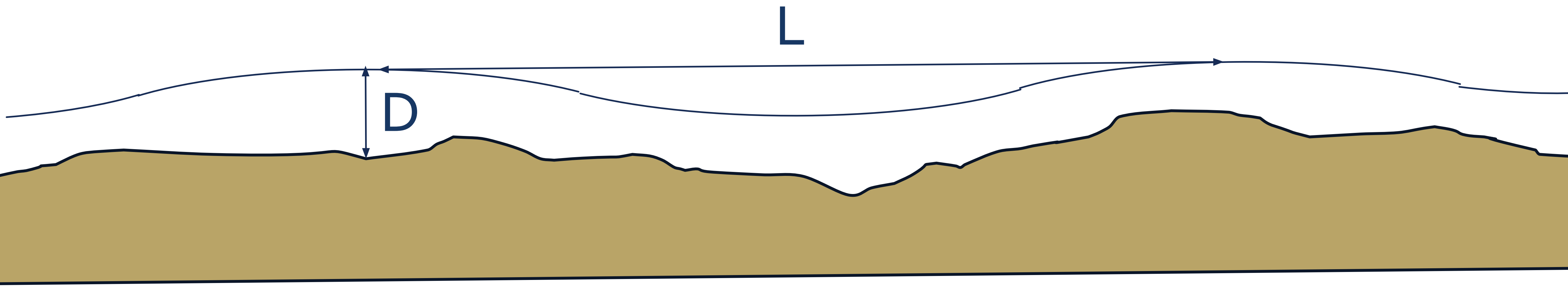
- AMPLITUDE - RANGE
- FRECUENCY – PERIOD - LENTH

→ ANALYSIS AND PREDICTIONS



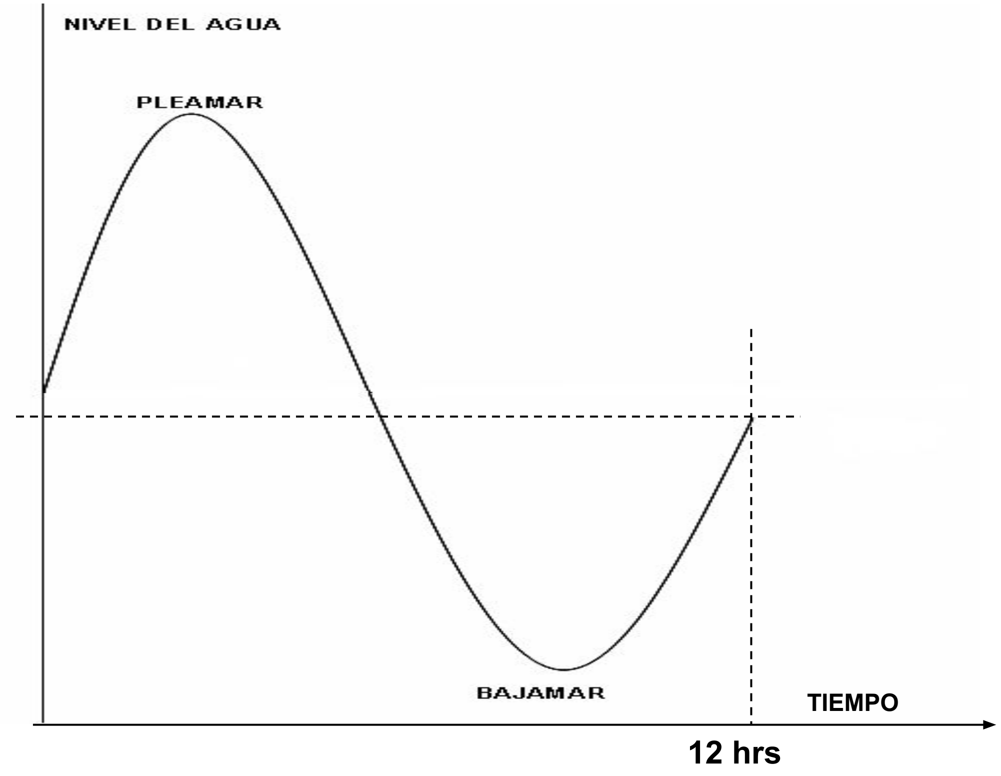


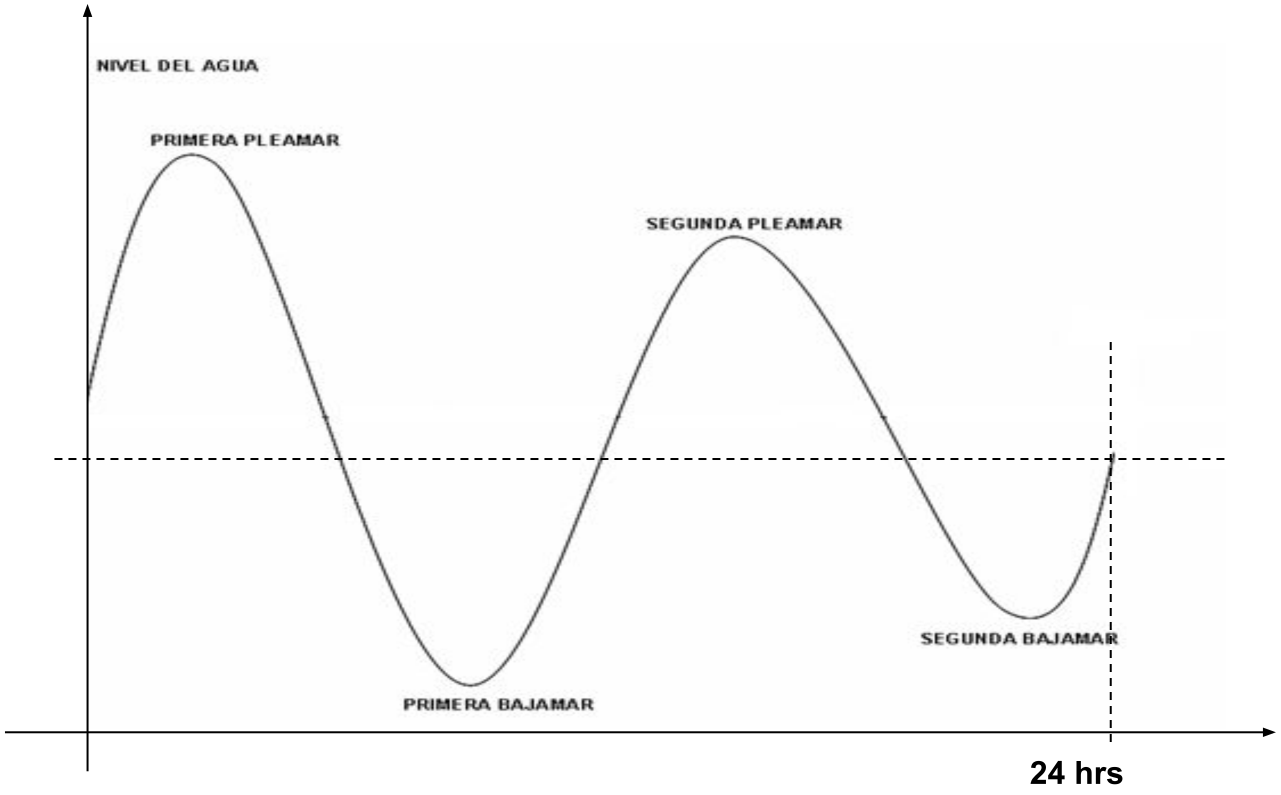
THE TIDE AS A LONG WAVE



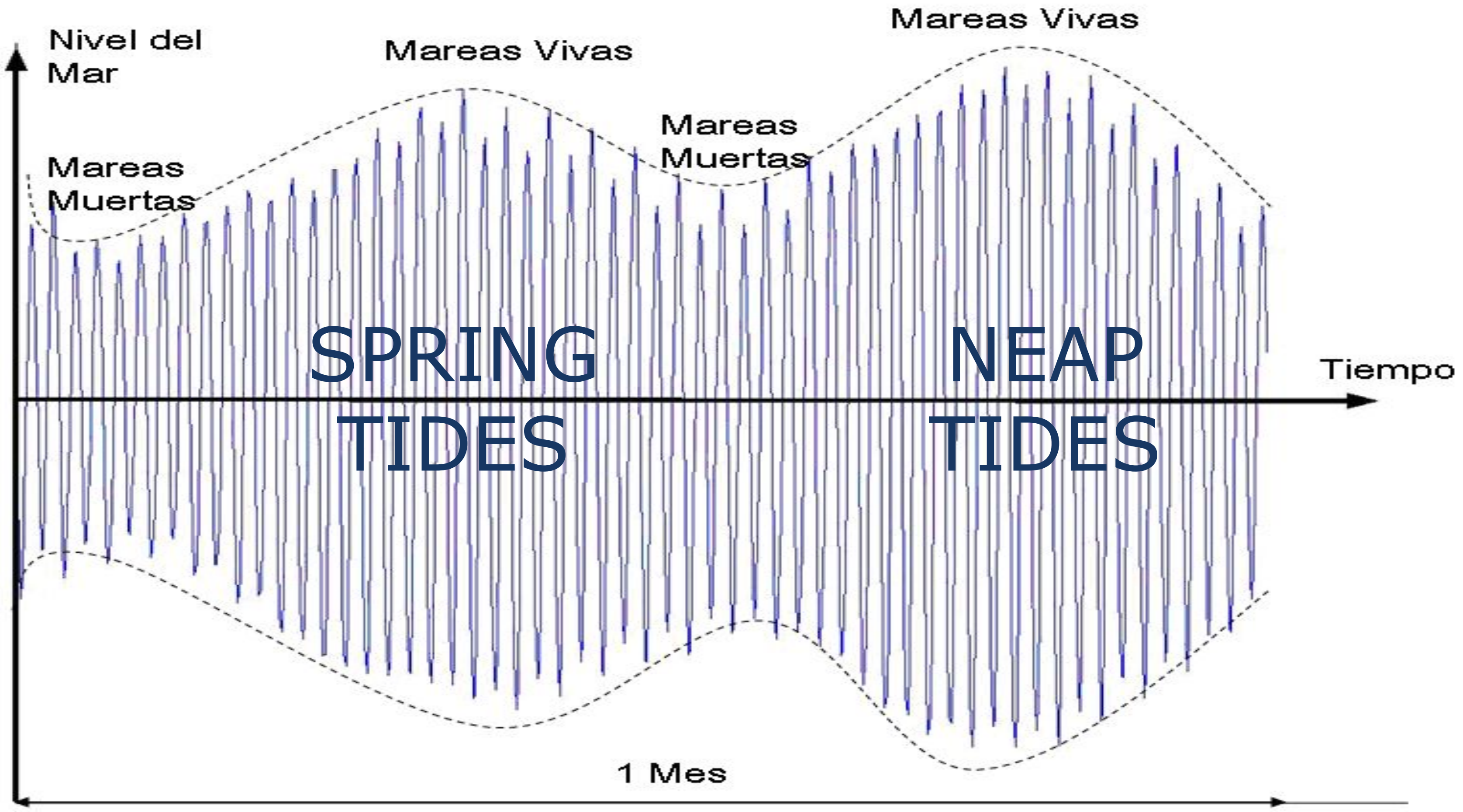
LENTH IS ›››››› THAN DEPTH

MORE DETAILS OF OUR WAVE…





MORE DETAILS OF OUR WAVE…



B) FUNDAMENTAL CONCEPTS OF EQUILIBRIUM THEORY

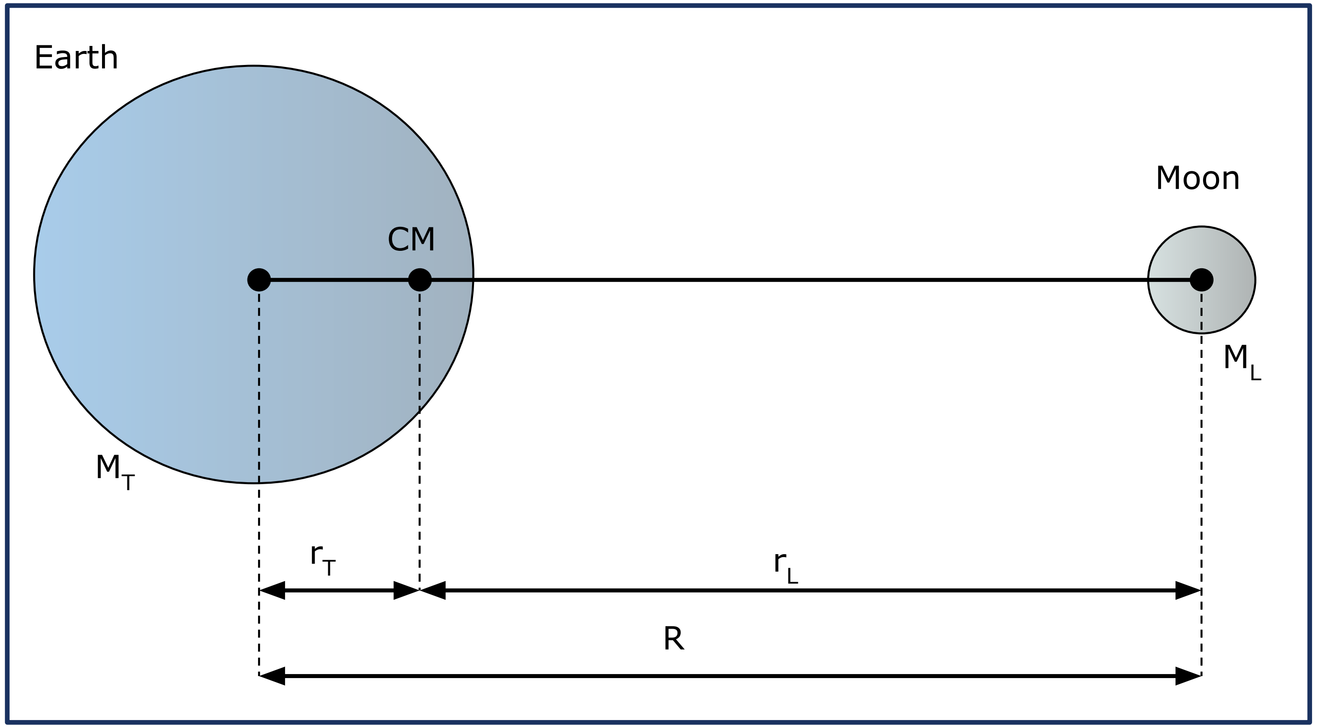
AN IDEAL WORLD

THE THEORY OF EQUILIBRIUM

According to the Theory of Equilibrium (**Newton** 1686), the free surface of the sea will assume a shape resulting from the **balance** of the forces involved in the **gravitational equilibrium** of the **Earth-Moon** system.

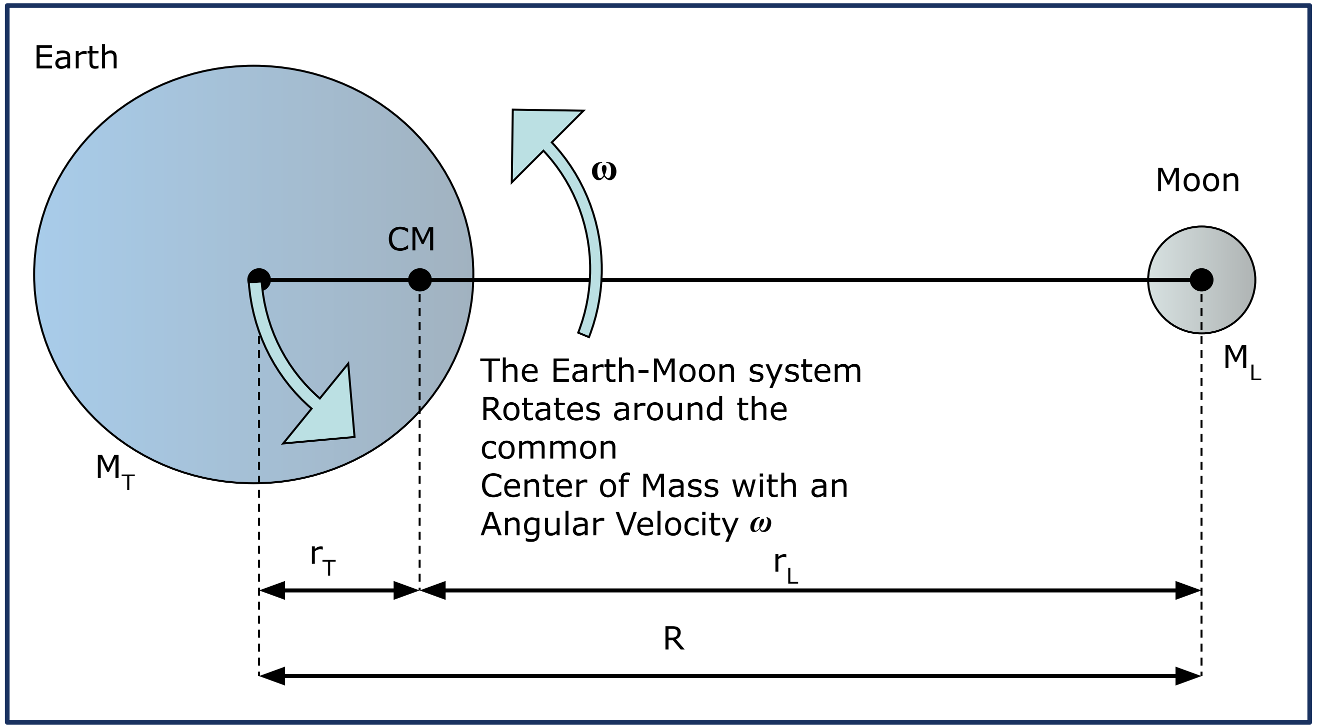
HYPOTHESIS

The only celestial bodies we will consider are the Earth and the Moon. These bodies revolve around their common center of mass.



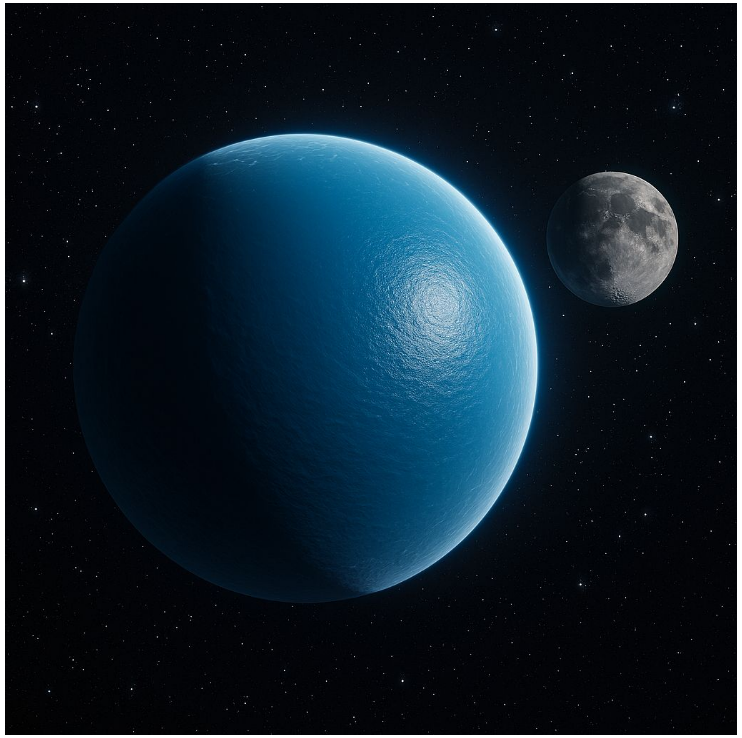
HYPOTHESIS

The rotation of the Earth-Moon system occurs in the plane of the Earth's Equator.



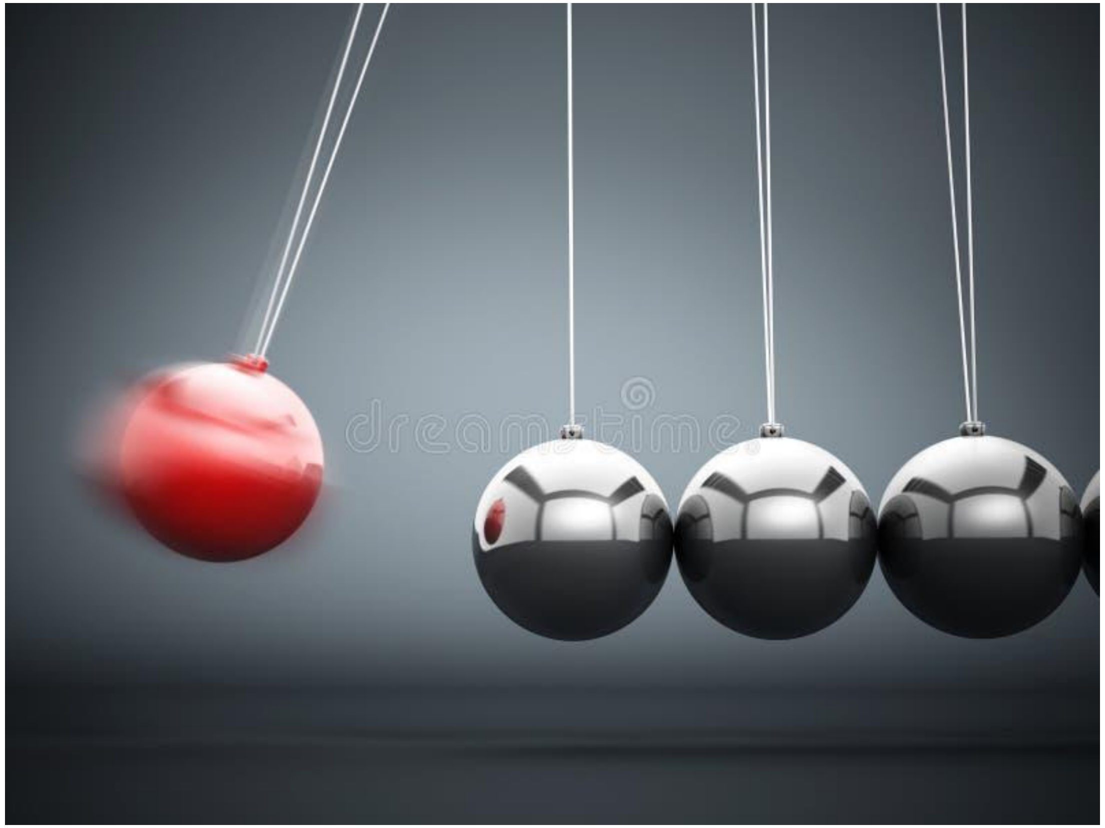
HYPOTHESIS

It is assumed that the Earth is perfectly spherical and uniformly covered by water. This hypothesis implies that there are no continents and that the depth of the single ocean is uniform.



HYPOTHESIS

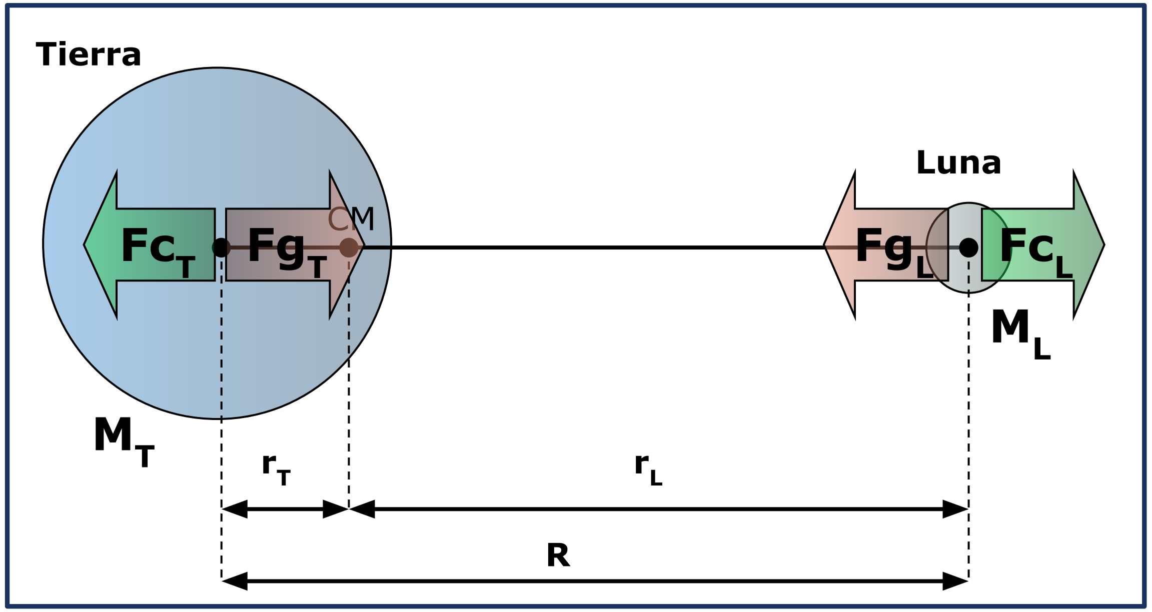
The body of water responds instantly to any force applied to it, meaning we consider that inertia does not exist and that the ocean's reaction is immediate.



C) THE EARTH AND THE MOON

A TALE OF ROMANCE AND DANCE

In the force equilibrium system shown in the figure, the forces acting on the Earth will be FcT and FgT, Centrifugal Force and Gravitational Attraction Force respectively, which will be equal in magnitude and opposite in direction:



r_T is the distance from the Earth`s center of mass to the common center of mass of the Earth-Moon system.
R is the distance from the Earth´s center of mass to the Moon`s center of mass.(**384329 km**).
G is the Universal Gravitational Constant (**$6{,}67428 \times 10^{-11}$ Nm^2kg^{-2}**).
M_T is the Earth`s mass (**$5{,}97 \times 10^{24}$ kg**).
M_L is the Moon`s mass (**$7{,}35 \times 10^{22}$ kg**).

$$Fc_T = M_T \omega^2 r_T$$

$$Fg_T = G\frac{M_T M_L}{R^2}$$

Since there is a balance of forces on the Earth and the Moon, we can assume that:

$$Fc_T = Fg_T$$

$$Fc_L = Fg_L$$

$$M_T \omega^2 r_T = G\frac{M_T M_L}{R^2}$$

$$M_L \omega^2 r_L = G\frac{M_T M_L}{R^2}$$

$$r_T = G\frac{M_L}{R^2\omega^2}$$

$$r_L = G\frac{M_T}{R^2\omega^2}$$

And given that:

$$R = r_T + r_L$$

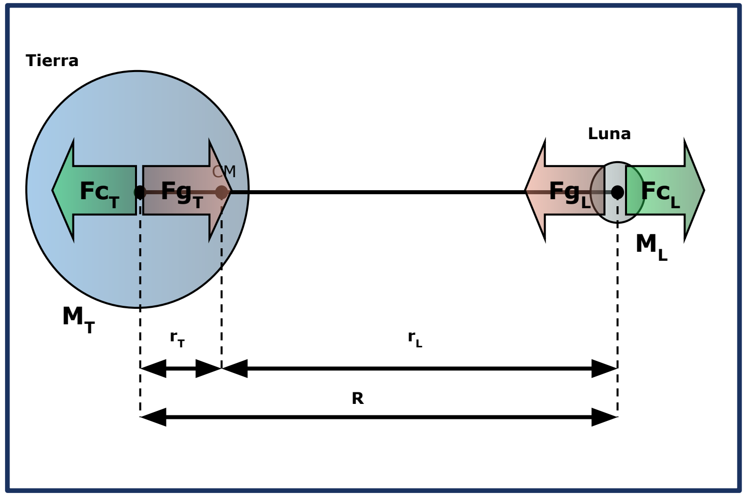
Operating balance:

$$R = G\frac{M_L}{R^2\omega^2} + G\frac{M_T}{R^2\omega^2}$$

$$R = \frac{G}{R^2\omega^2}(M_L + M_T)$$

$$\omega^2 = \frac{G}{R^3}(M_L + M_T)$$

$$\omega = \sqrt{\frac{G}{R^3}(M_L + M_T)}$$



$$\omega = \sqrt{\frac{G}{R^3}(M_L + M_T)}$$

$$\omega = \sqrt{\frac{6.67428 * 10^{-11}}{(384329 * 10^3)^3}(7.35 * 10^{22} + 5.97 * 10^{24})}$$

$$\omega = 2.66558 * 10^{-6} rad/seg$$

$$r_T = G\frac{M_L}{R^2\omega^2}$$

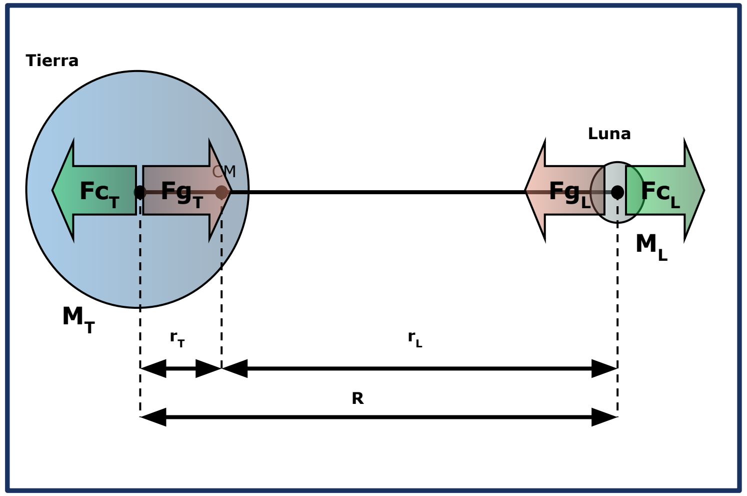
$$r_T = 6.67428 * 10^{-11}\frac{7.35 * 10^{22}}{(384329 * 10^3)^2 x(2.66558 * 10^{-6})^2}$$

$$r_T = 4674143\ m$$

$$r_L = R - r_T$$

$$r_L = 384329000 - 4674143$$

$$r_L = 379654857\ m$$

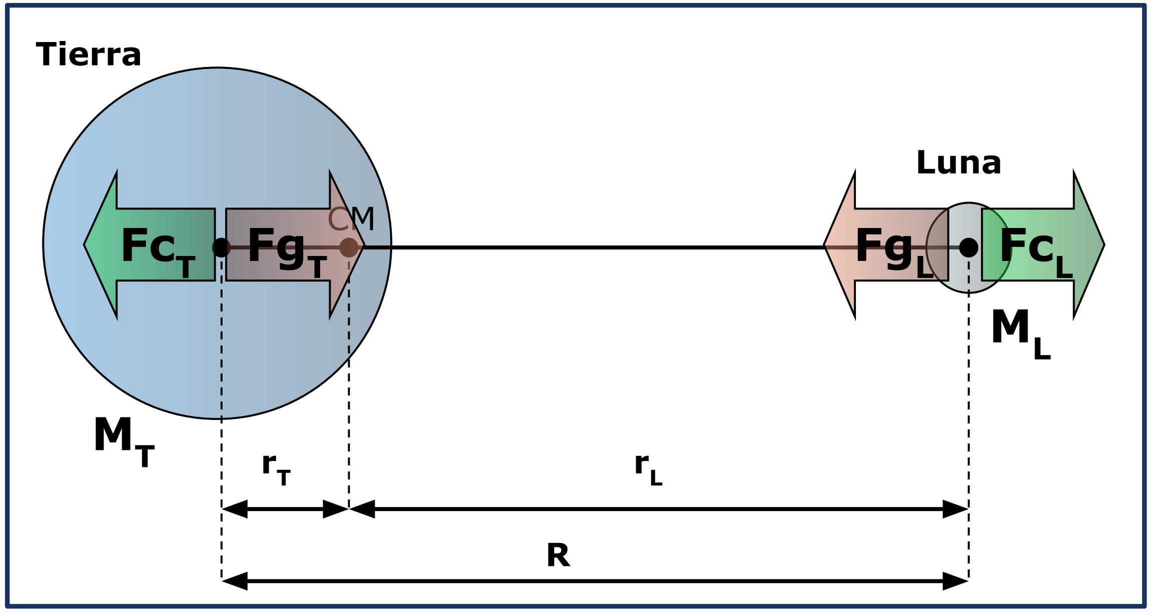


THE CENTER OF MASS IS INSIDE THE EARTH

The value obtained for the angular velocity of the Earth-Moon system is $\boldsymbol{\omega}$ **= 2,66558 x 10^{-6} rad/sec**. It is useful to determine, from this result, the duration of that orbit. That is, the period *T* of this rotation. Given that one orbit corresponds to 2ϖ radians, those 2ϖ radians will be covered in a period *T* and we have:

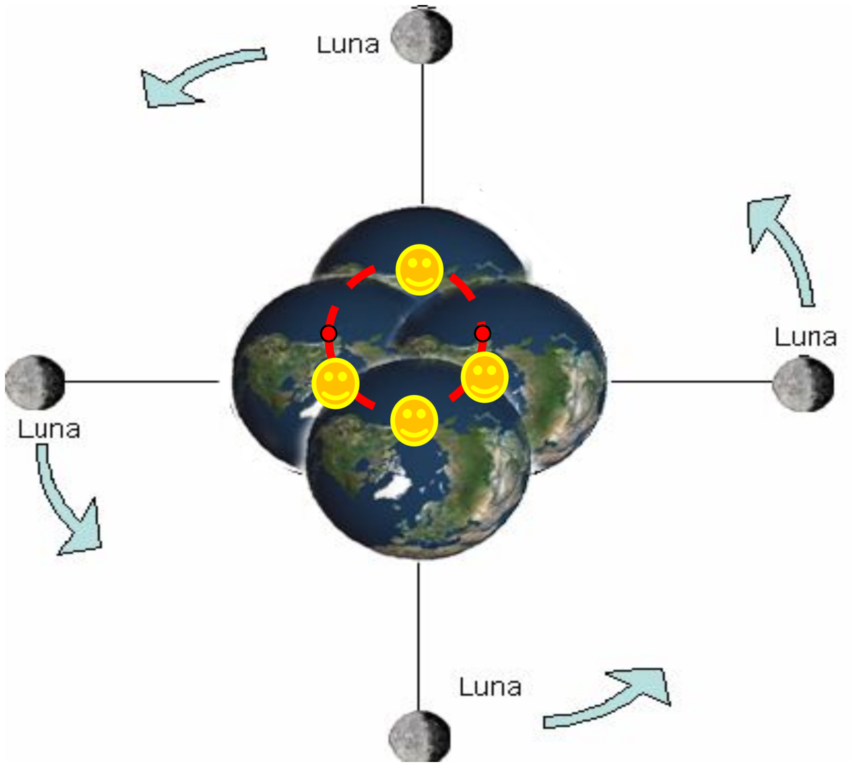
$$\omega = \frac{2\pi}{T} \leftrightarrow T = \frac{2\pi}{\omega}$$

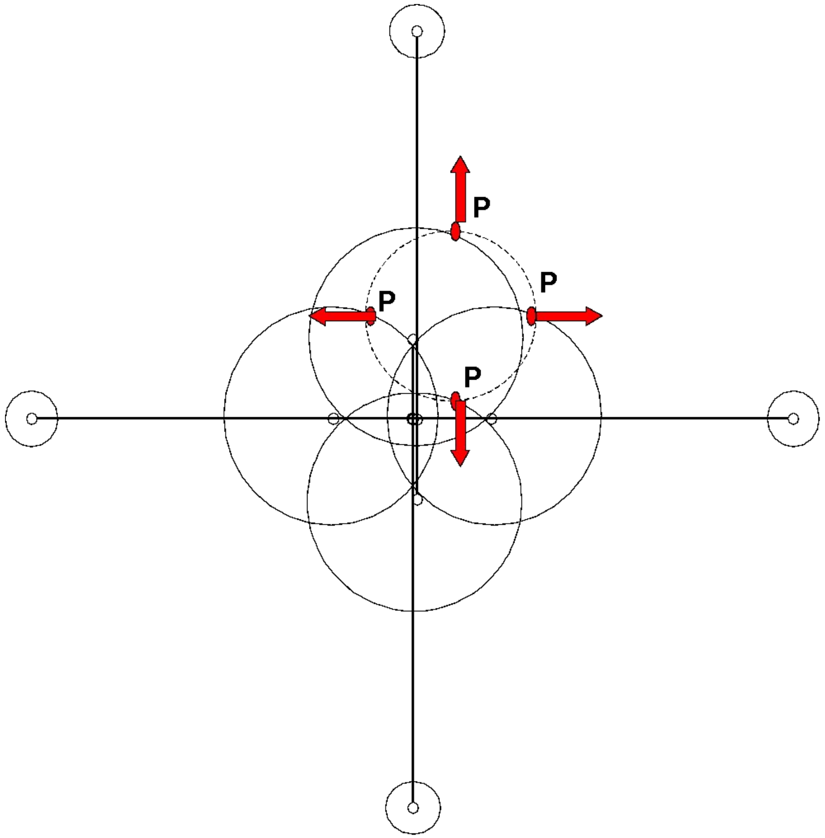
$$T = \frac{2\pi}{2.66558 * 10^{-6}} = 2357155\, seg = 27.28\, dias$$



That is to say, the Earth-Moon system completes an orbit in **27.3 days**, which is equivalent to a **sidereal month**.

On the other hand, if we observe the movement of the Earth-Moon system **and focus on a point on the Earth's surface (P),** we can see that the path followed by that point is a rotation with the **same radius as that described by the center of the Earth**, that is, $\boldsymbol{r_T}$.



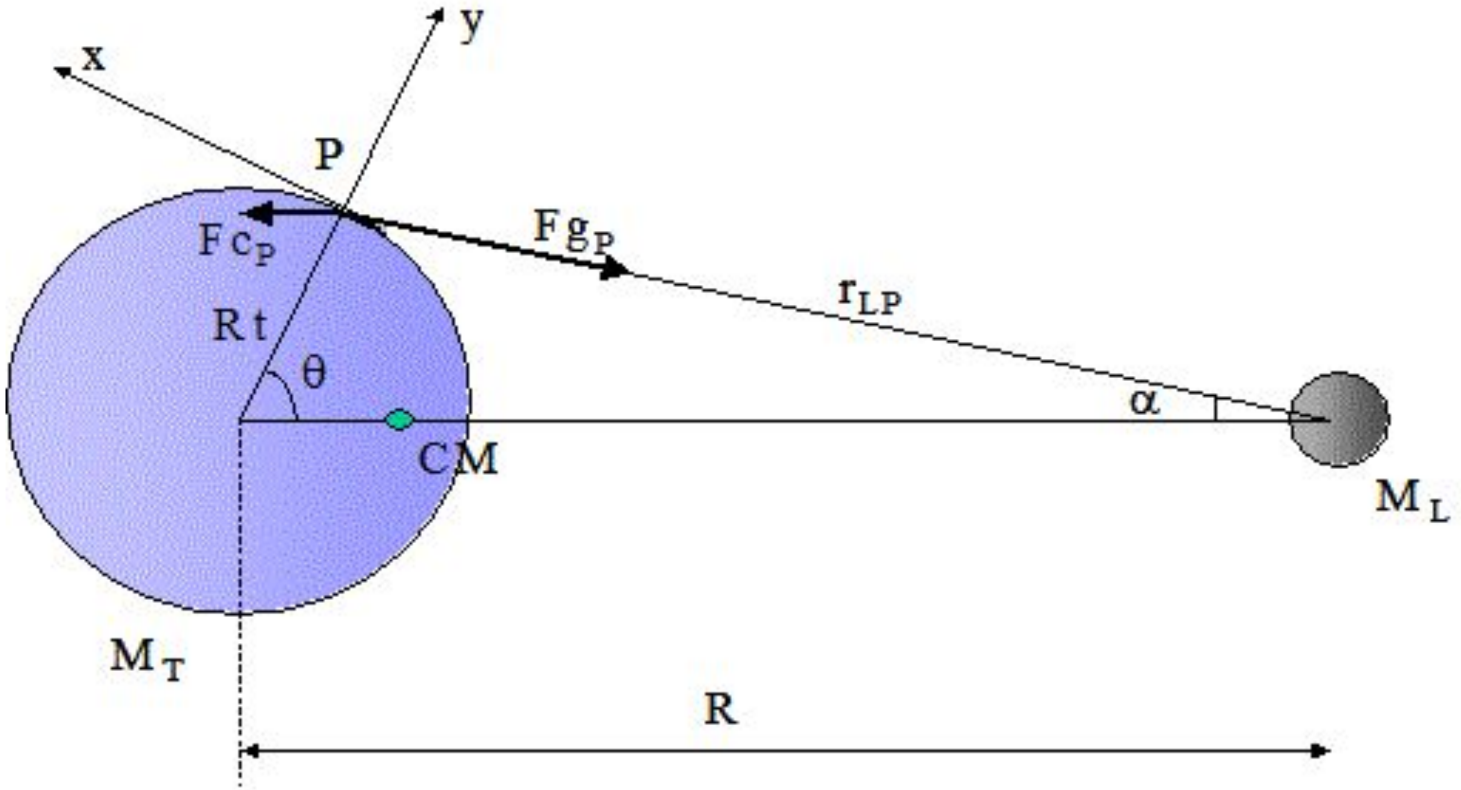


$$Fc_P = \omega^2 r_t$$

$$Fg_P = G\frac{M_L}{R^2}$$

This **centrifugal force**, as can be seen in the figure, will be directed at each moment in **a direction parallel to the line connecting the centers of mass of the Earth and the Moon and opposite to the latter**.

In the formula, we can see that the value of the gravitational attraction force $\boldsymbol{Fg_P}$ depends on the distance $\boldsymbol{r_{LP}}$ from point P to the Moon. Therefore, the value of this gravitational attraction force will be different for various points on the Earth's surface.

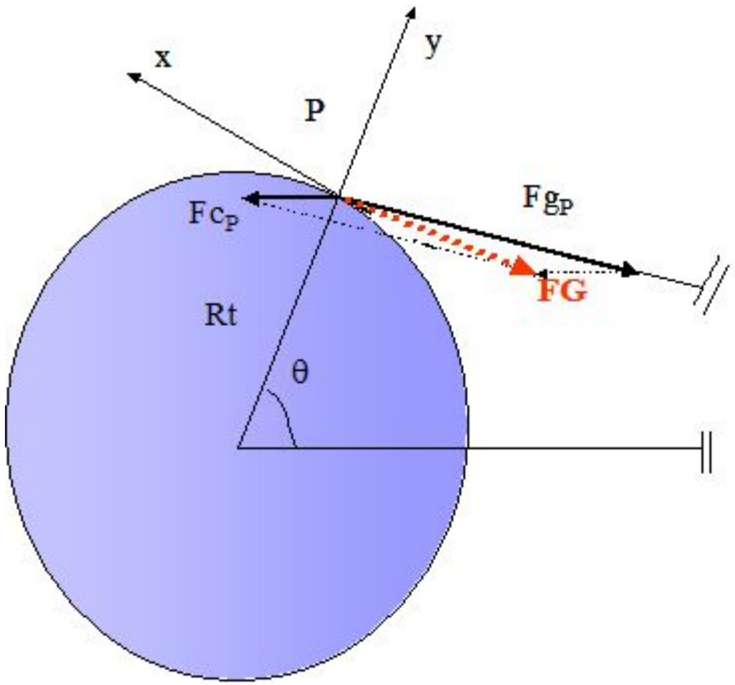


$$Fg_P = G\frac{M_L}{r_{LP}^2}$$

The **centrifugal force**, as can be seen in the figure, will be directed at each moment in **a direction parallel to the line connecting the centers of mass of the Earth and the Moon and opposite to the latter**.

On the contrary, we had observed that the centrifugal force $\boldsymbol{Fc_P}$ at that same point P was always the same and did not vary when choosing a new position on the Earth.

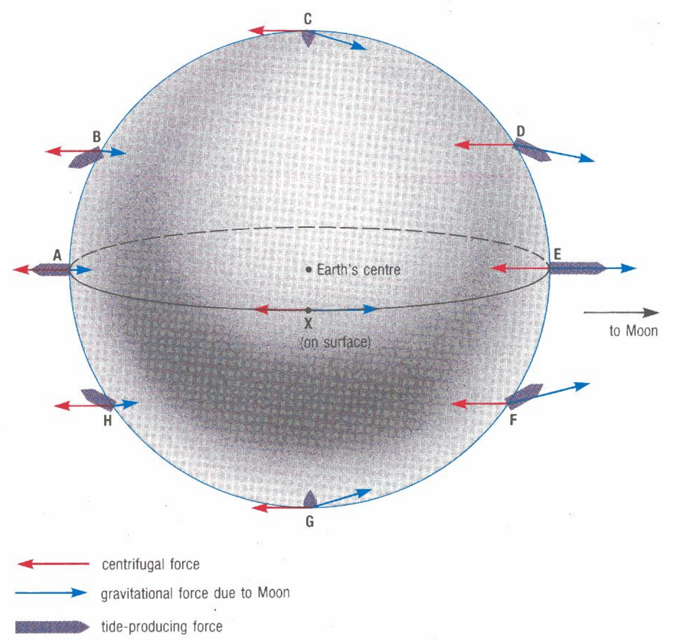
The resultant of both forces $\boldsymbol{Fc_P}$ and $\boldsymbol{Fg_P}$ for each point on the Earth's surface is called the **Tide Generating Force *FG***.

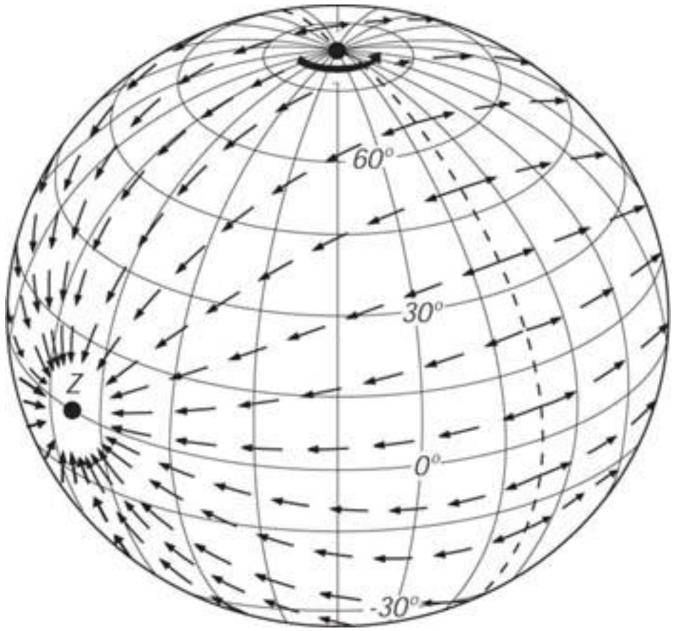


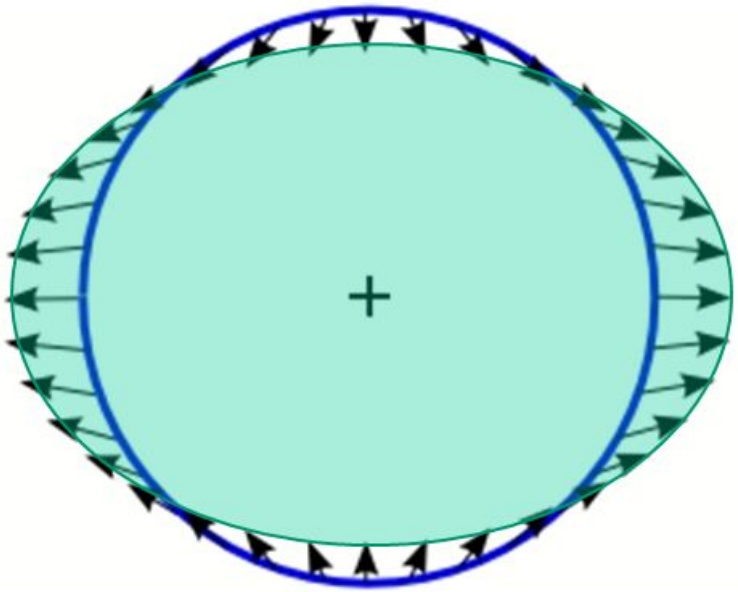
The **Tide Generating Force FG at each point P** will therefore be the part of the Gravitational Attraction Force caused by the Moon that is not compensated by the Earth's Centrifugal Force.

The horizontal component of the resultant will be called the **TIDAL TRACTIVE FORCE**. After performing certain calculations and applying some geometric simplifications, the value of the Tractive Force will be:

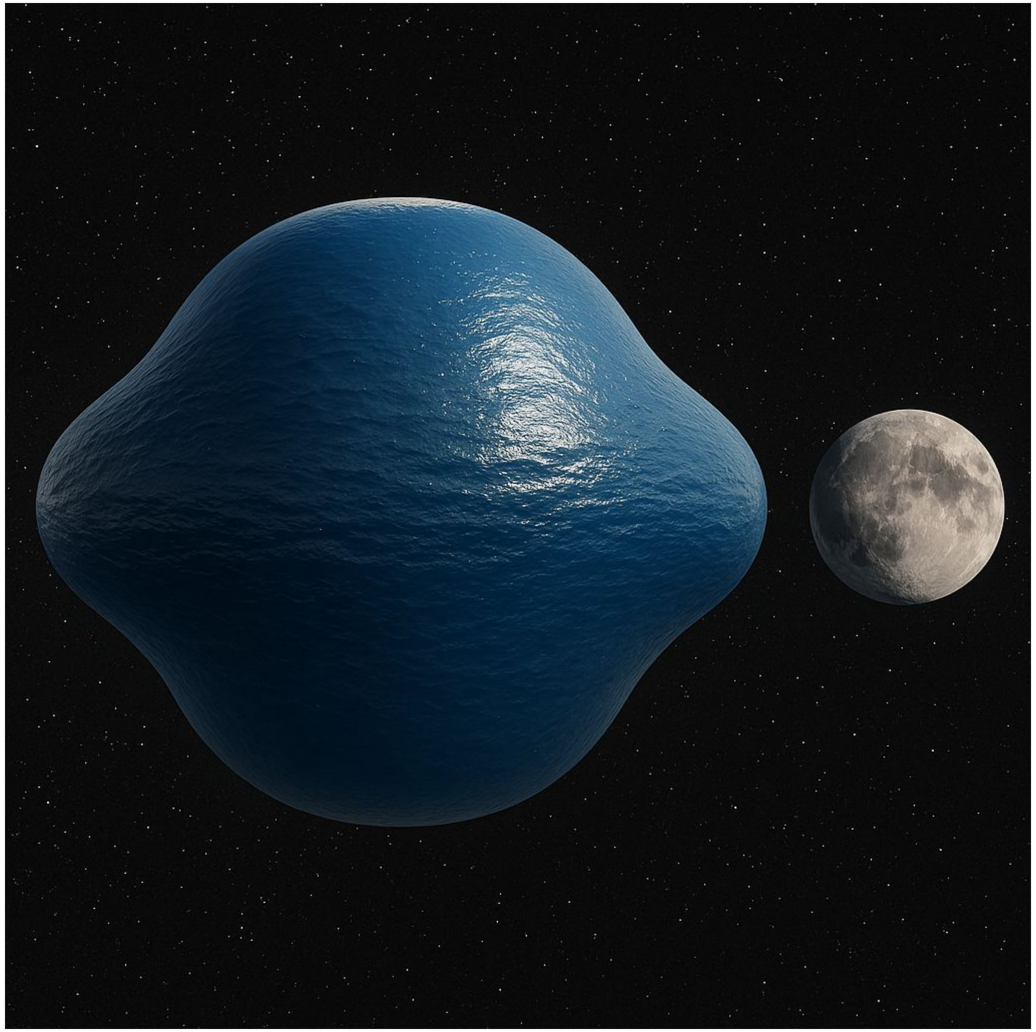
$$Ft = GM_L \frac{3R_T \sin 2\theta}{2R^3}$$







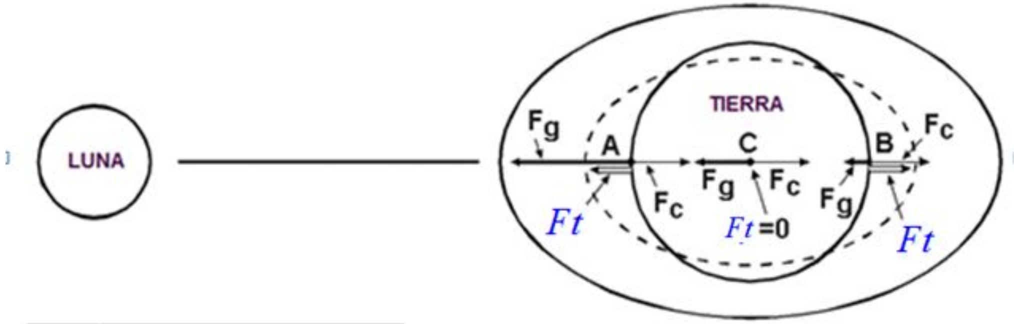
EQUILIBRIUM ELLIPSOID



F_c = FUERZA CENTRÍFUGA

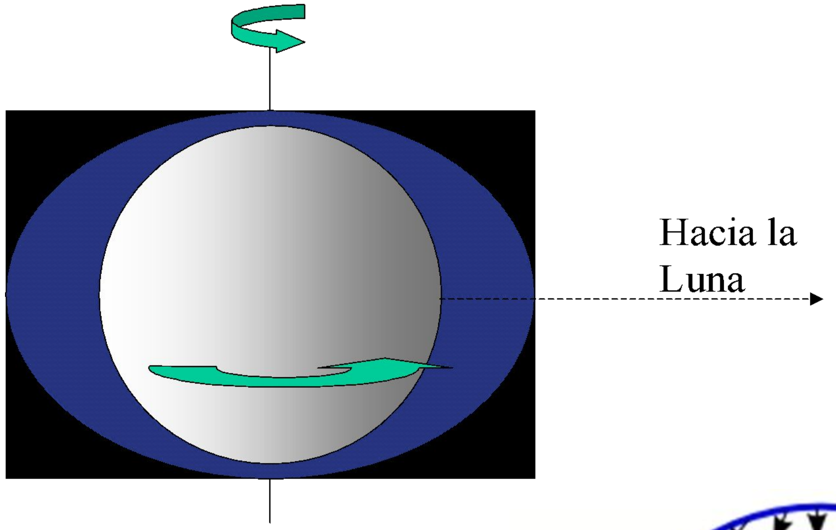
F_g = FUERZA DE GRAVEDAD

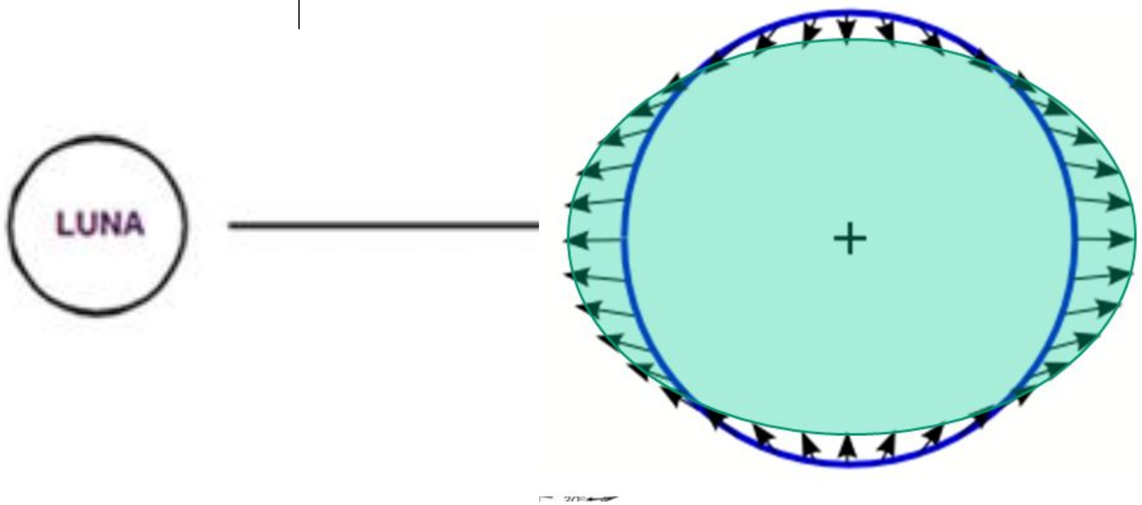
Ft = RESULTANTE

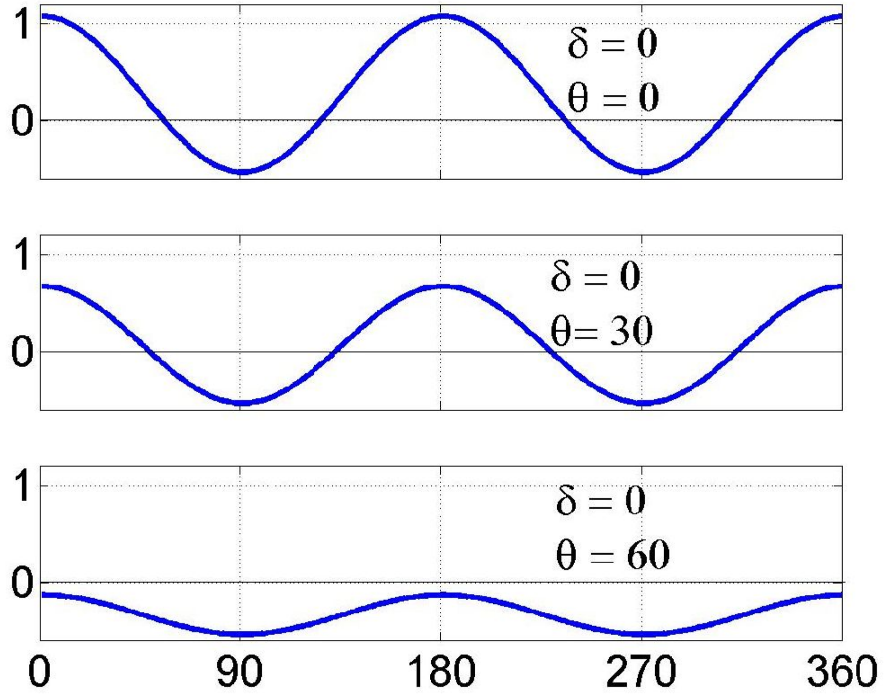


A	$F_g > F_c$	Ft
C	$F_g = F_c$	0
B	$F_g < F_c$	Ft

Let's add THE EARTH'S ROTATION

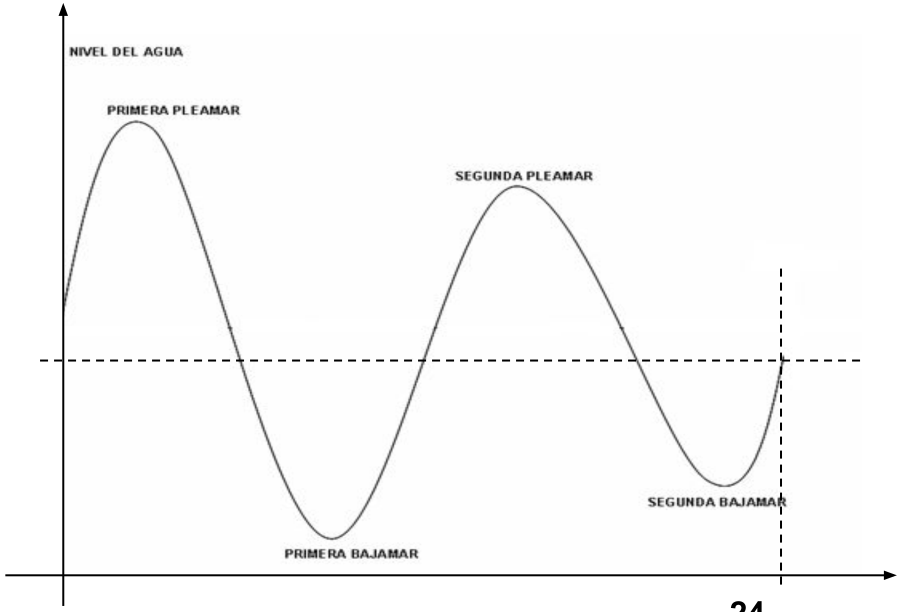






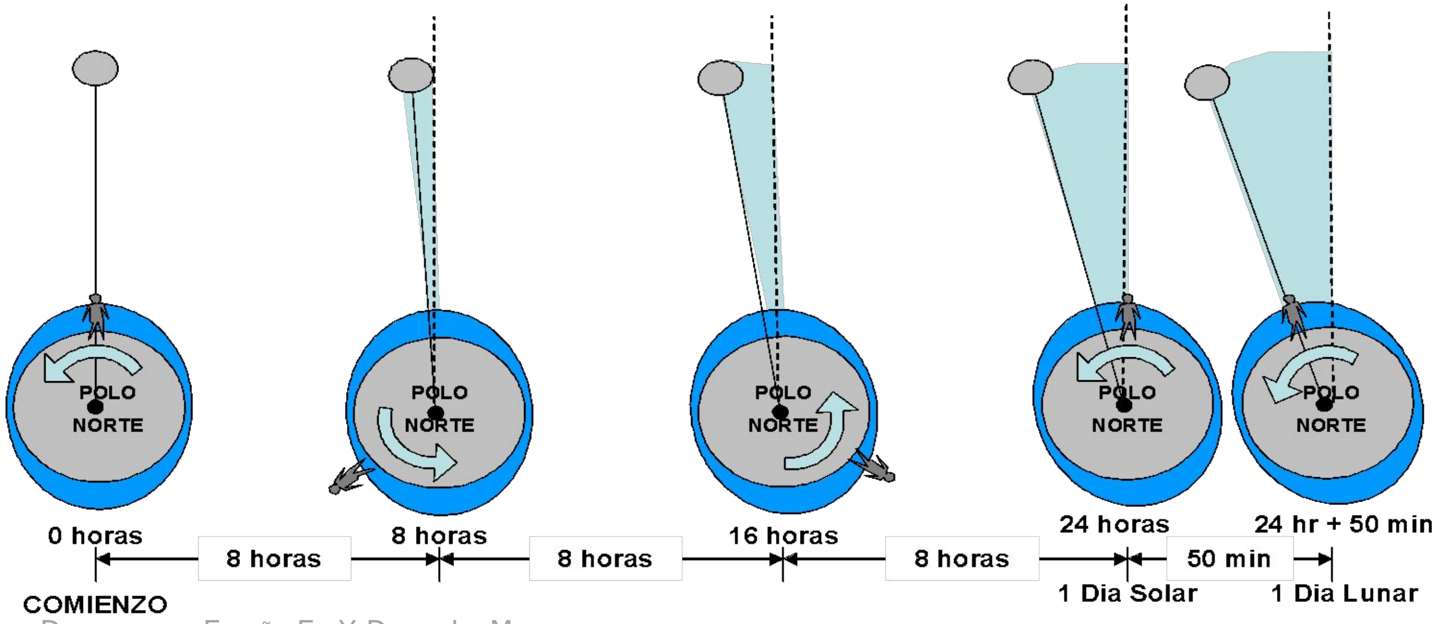
SEMIDIURNAL TIDES

Why does it take more than 24 hours?

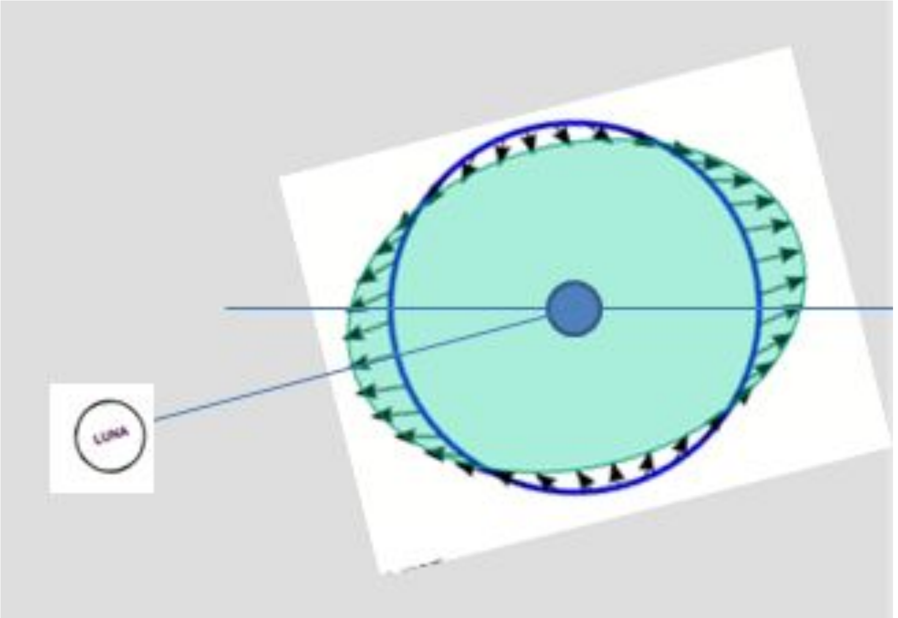


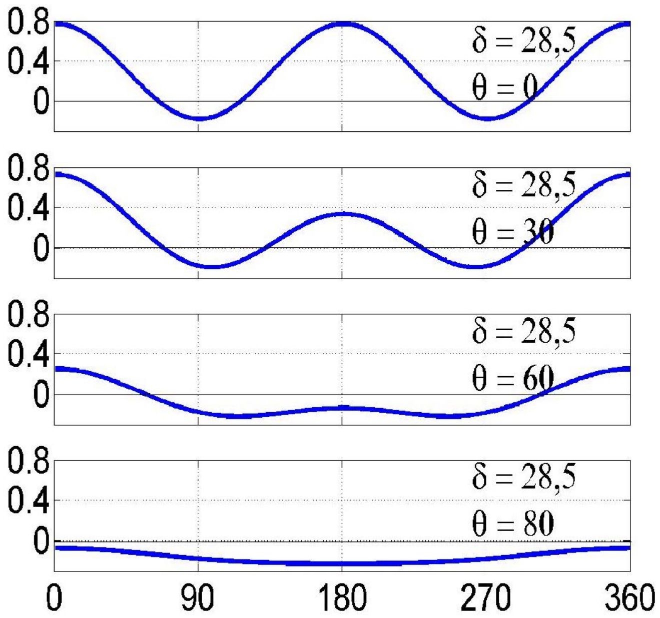
In that figure, we started with the Moon passing over our meridian. Being in the vertical position of the Moon means we are at a bulge, or in other words, we will experience high tide. From that moment on, the Earth will continue its rotation. As we move along with the Earth's rotation, the Moon also moves because it is orbiting around the center of mass of the Earth-Moon system.

As we can recall, at the beginning we explained that if we traced **two complete tidal cycles**, we observed that the duration of these two cycles was approximately **24 hours and 50 minutes**.



Let`s add a new level: THE LUNAR DECLINATION (-28.8º AND +28.8º)





DIURNAL INEQUALITY

DIURNAL TIDES
AT HIGH LATITUDES

C) THE SUN

A FRESH COMPANION JOINS THIS PAIR.

The periodicity of the tides caused by the Sun is also easier to determine.

Similarly to what happens with the Moon, the gravitational force caused by the Sun will also produce its corresponding tidal ellipsoid. This ellipsoid will have its bulges permanently pointing towards the Sun and the point diametrically opposite.

As a result, **the Sun will also produce semidiurnal tides**.

There will be a "solar high tide" each time the Sun passes directly overhead or at the point diametrically opposite. Therefore, **there will be two solar high tides every 24 hours exactly** (the length of the solar day); **in other words, one solar high tide every 12 hours**.

Just as with the Moon's declination, the **23.5º** tilt of the **Ecliptic** (the plane of Earth's orbit around the Sun) causes diurnal inequalities in solar tides, as well as diurnal solar tides at high latitudes.

WHICH IS MORE POWERFUL? THE SUN OR THE MOON?

$$Ft_L = GM_L \frac{3R_T \sin 2\theta}{2R_L^3}$$

$$Ft_S = GM_S \frac{3R_T \sin 2\theta}{2R_S^3}$$

$$\frac{Ft_S}{Ft_L} = \frac{GM_S \dfrac{3R_T \sin 2\theta}{2R_S^3}}{GM_L \dfrac{3R_T \sin 2\theta}{2R_L^3}}$$

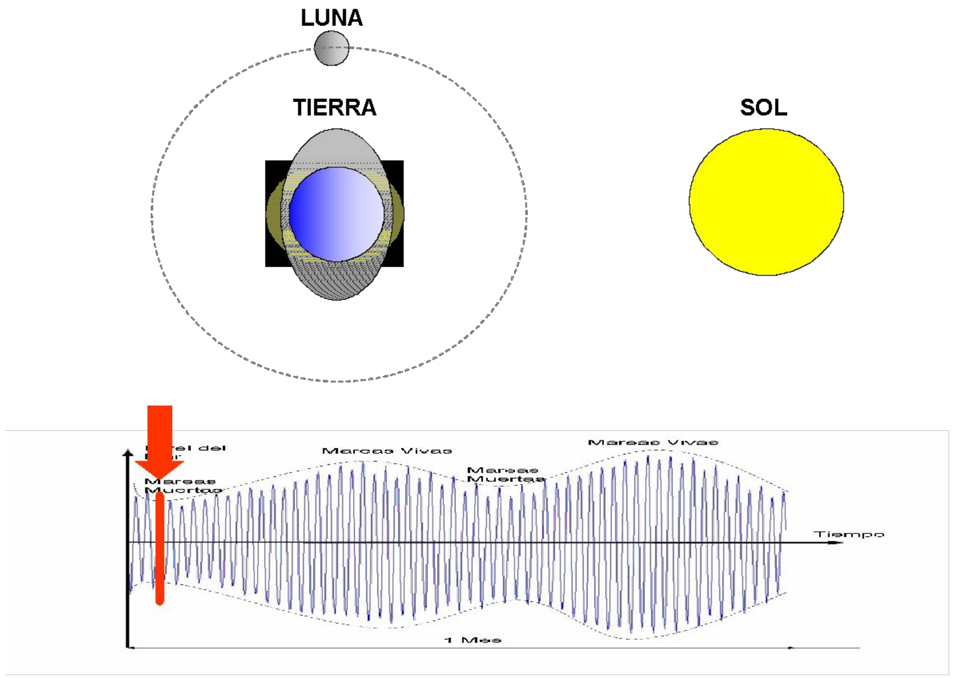
$$\frac{Ft_S}{Ft_L} = \frac{M_S R_L^3}{M_L R_S^3} = \frac{(1.9891x10^{30})x(3.84329x10^{8})^3}{(7.35x10^{22})x(1.49597871x10^{11})^3}$$

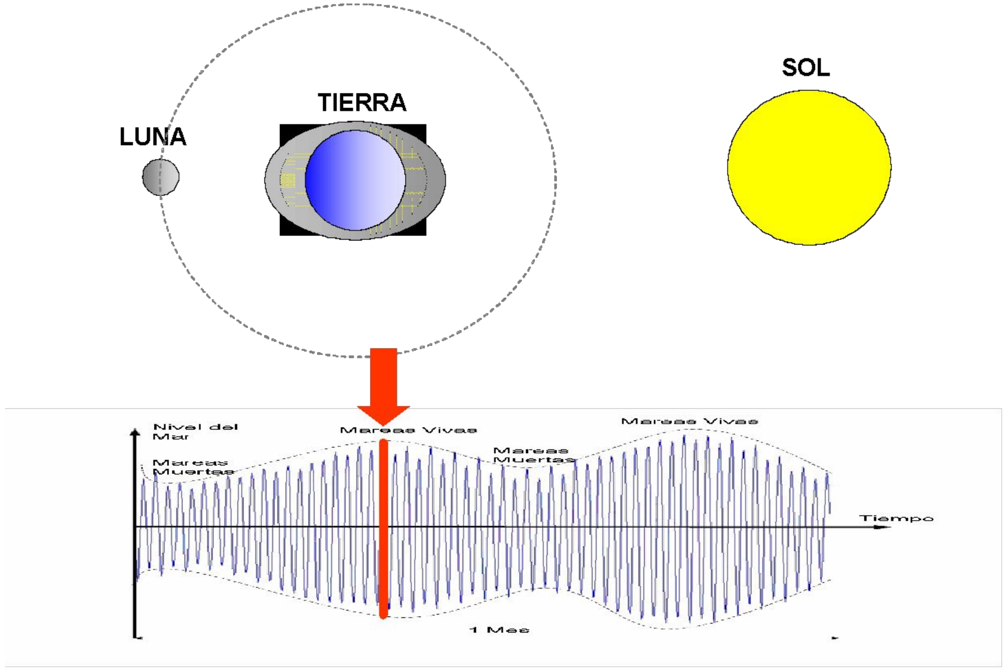
$$\frac{Ft_S}{Ft_L} = 0.46$$

D) EARTH – MOON - SUN

A TRIO IS BETTER THAN A PAIR

THE COMBINATION OF FORCES IS THE SUM OF THEM

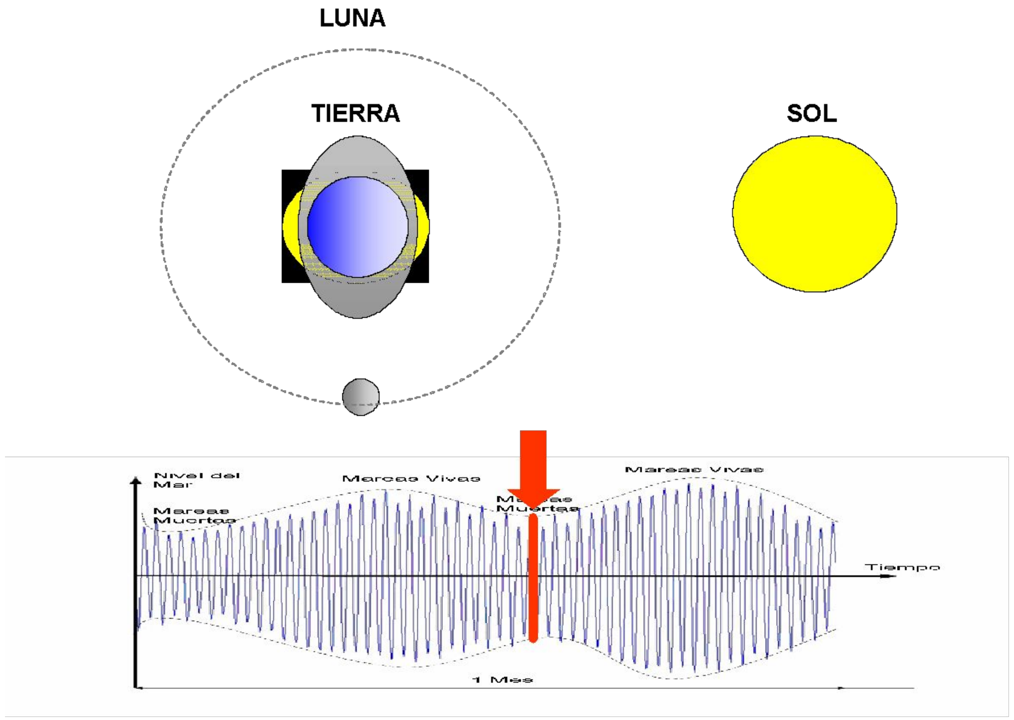




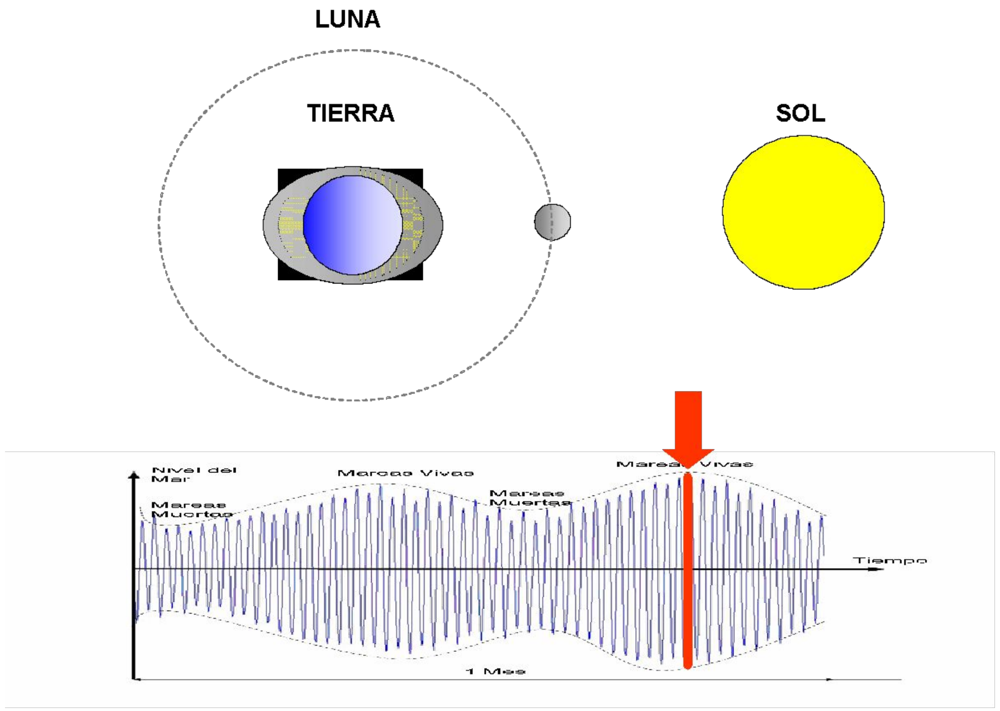
FRIST QUARTER MOON
NEAP TIDES

FULL MOON
SPRING TIDES

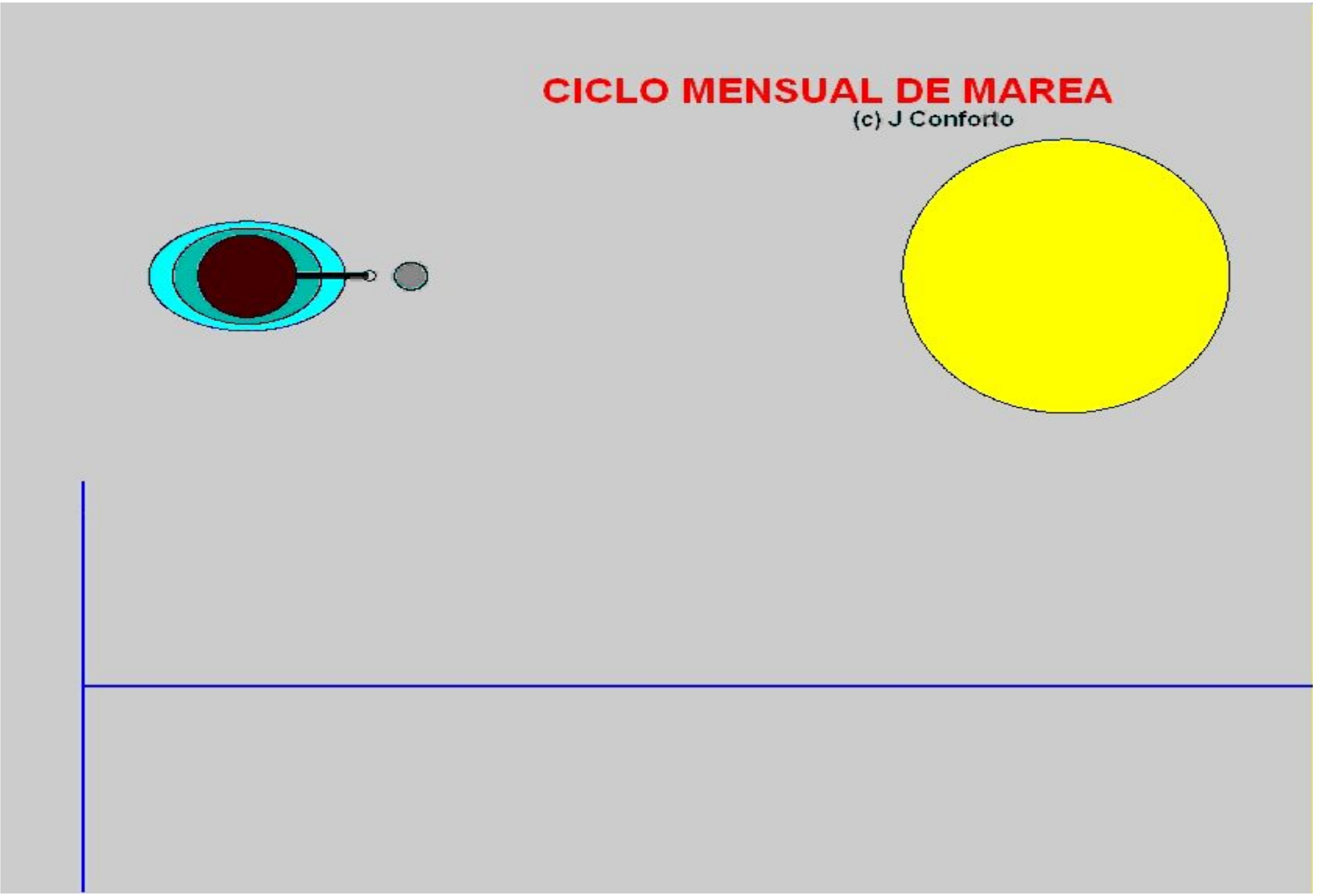
THE COMBINATION OF FORCES IS THE SUM OF THEM



WANNING QUARTER MOON
NEAP TIDES



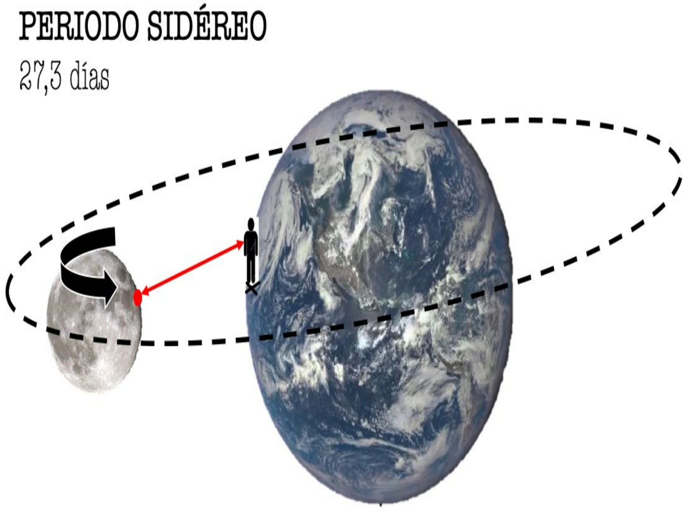
NEW MOON
SPRING TIDES

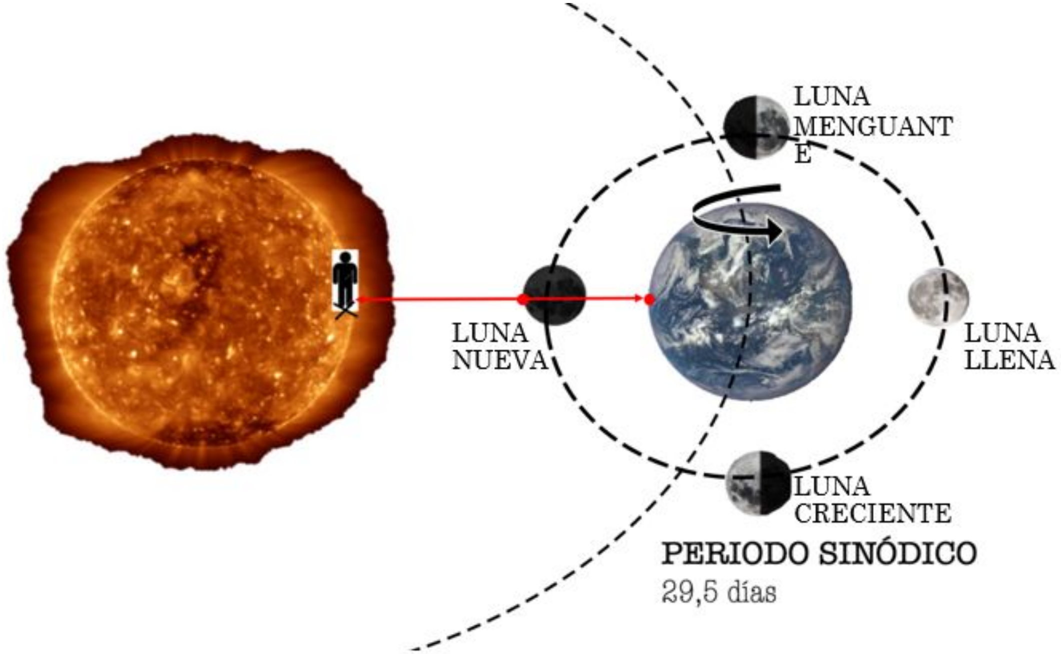

CICLO MENSUAL DE MAREA
(c) J Conforto

Let's add another level, the MOON'S ELLIPTICAL ORBIT.

- PERIGEE – APOGEE – ANOMALISTIC MONTH (27.554 DAYS)

The result is that each month the perigee occurs with a different phase of the Moon, completing a cycle after **8.85 years**.





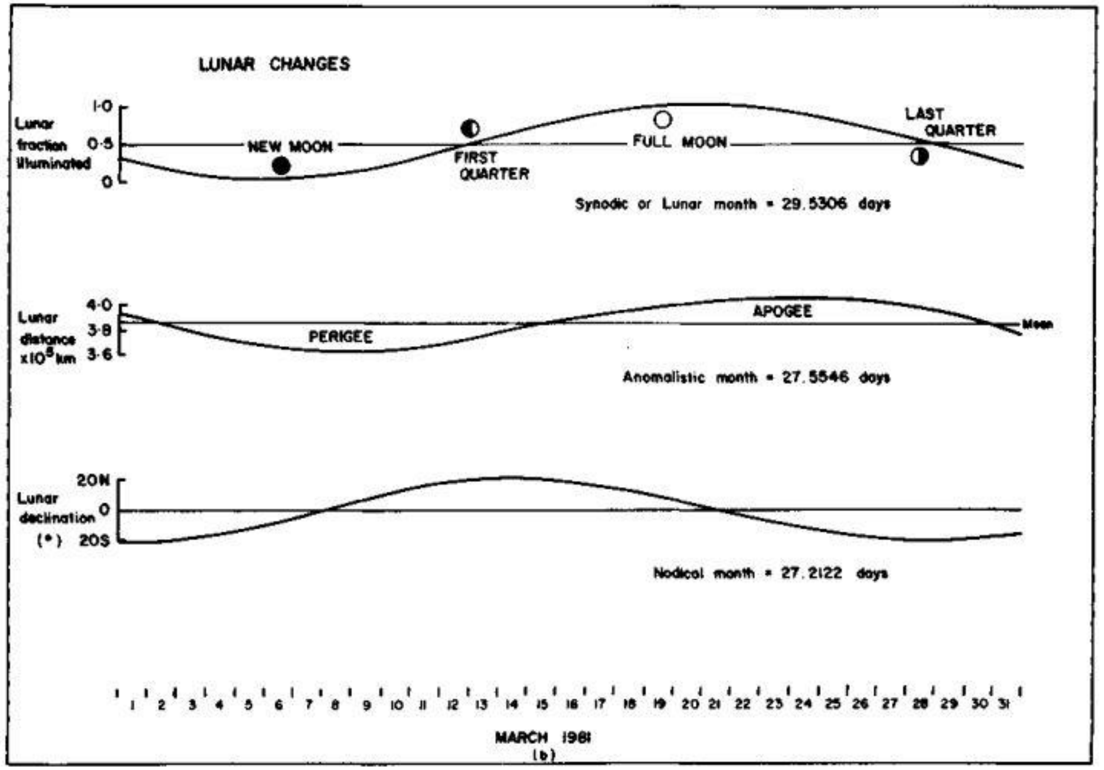
Another level: THE EARTH'S ELLIPTICAL ORBIT AROUND THE SUN

- PERIHELION – APHELION – ANOMALISTIC YEAR (365,2596 DAYS)

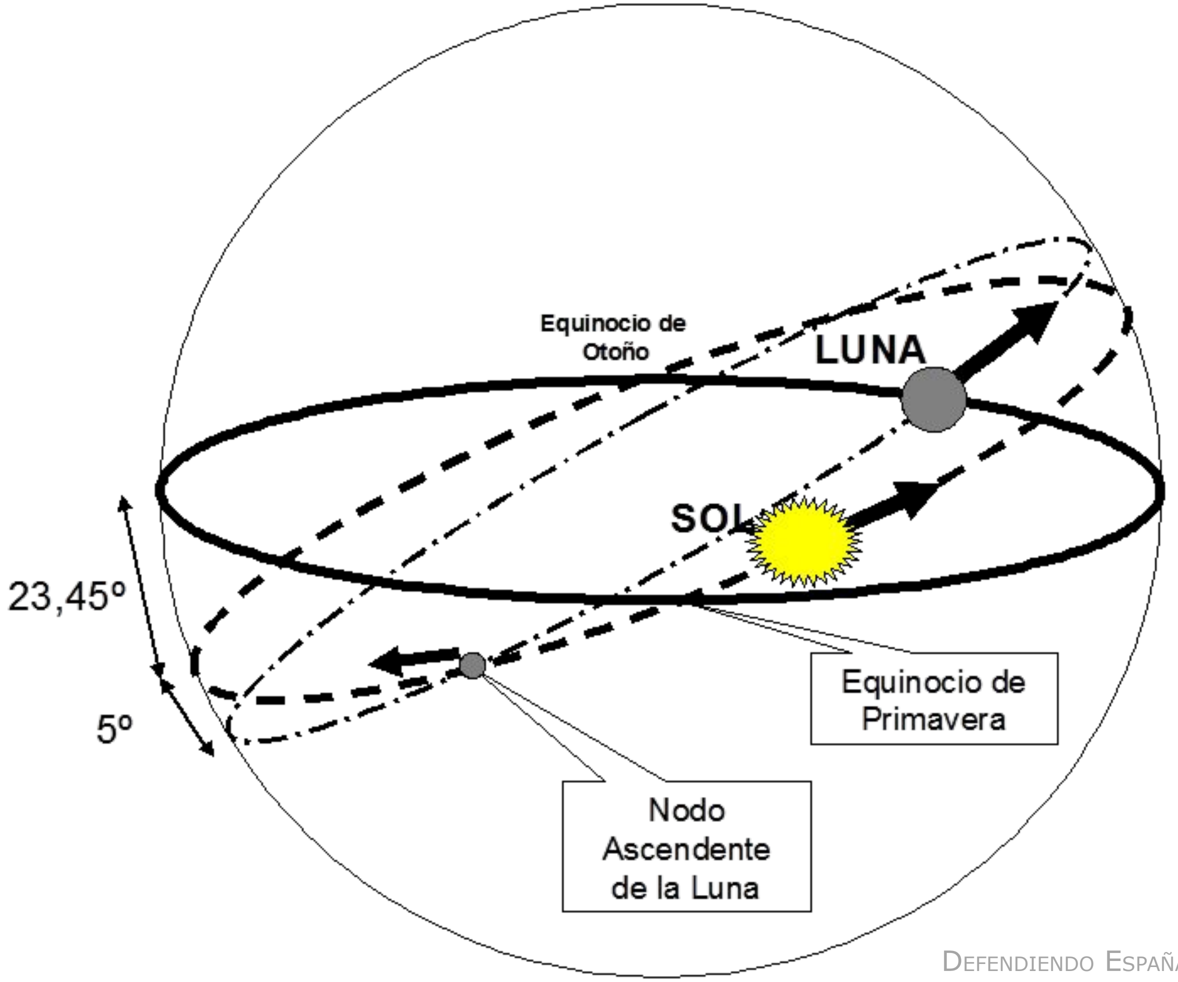
AND THE FINAL LEVEL, THE DECLINATION OF THE EARTH'S ORBIT TO THE SUN'S EQUATORIAL PLANO (23,4º).

Spring and autumn equinoxes.

Description	Frequency notation (1/period)	Period (mean solar units)
Sidereal day (one rotation wrt vernal equinox)	Ω	23.9344 hours
Mean solar day (one rotation wrt to the sun)	ω_s	24.0000 hours
Mean lunar **day** (one rotation wrt to the moon)	ω_L	24.8412 hours
Period of lunar declination (tropical **month**)	ω_1	27.3216 days
Period of solar declination (tropical **year**)	ω_2	365.2422 days
Period of lunar perigee	ω_3	8.847 years
Period of lunar node	ω_4	18.613 years
Period of perihelion	ω_5	20,940 years



Another cyclical process we must bear in mind comes from the fact that the planes of the Earth's orbit around the Sun and the Moon's orbit around the Earth are inclined relative to each other. Both planes intersect on the celestial sphere at two points, called the **Moon's Ascending Node and the Moon's Descending Node.**



These points are not fixed because one plane continuously rotates relative to the other. If we take the **Moon's Ascending Node** as a reference on the Ecliptic, we can see that this point has a retrograde motion along the Ecliptic that completes over **18.61 years**. This is called the **Nodal Cycle**. These different inclinations of the planes of both orbits will produce **increases and decreases in tidal amplitude** over their 18.61-year cycle, which are referred to as the Nodal Tide.

OBSERVATIONS ON THE EQUILIBRIUM THEORY

Theoretically, according to the equilibrium theory, high tides should occur **at the moment** the Moon or the Sun passes through the local meridian. However, observations of the real world show us that this is not the case. In fact, spring tides occur between **one and three days after** the New Moon or Full Moon, that is to say, they have a significant delay.

This could be expected from the moment we assumed in our equilibrium theory that there was no inertia in the motion of the ocean and that movements would occur immediately when the causing force acted. However, the enormous mass of the oceans gives them, in reality, **tremendous inertia**, which produces significant delays.

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